

Scottish Survey of Literacy and Numeracy (SSLN)

Professional Learning Resource

Numeracy and
Mathematics Skills



Transforming lives through learning

Numeracy and Mathematics Skills

Progression of skills underpins the philosophy of Curriculum for Excellence. Numeracy and mathematics skills are embedded in the experiences and outcomes and cannot be taught in isolation. Skills in numeracy and mathematics can be developed through the planning of activities, questions and assessment which encourage learners to think about the concepts, going beyond recall of knowledge and encouraging exemplification of understanding.

The analysis of Scottish Survey of Literacy and Numeracy results identified key skills required to support and extend knowledge and understanding in numeracy and mathematics. One of the key areas for development identified from analysis of learners' responses in the survey was learners' ability to apply their skills.

In numeracy and mathematics there is a myth that there is always a right answer and a right method for every problem. In real life, there is usually no such thing. What TV should I buy? Which bank account should I open? Where should we go on holiday? Reasoning in numeracy and mathematics are essential to helping us make informed choices, and to be able to explain to others how they came about.

This document supports exemplification of skills, as detailed in the [Assessing Progress and Achievement in numeracy and mathematics progression framework](#); they should not be developed in isolation. Practitioners should ensure that skills are applied across a range of contexts and significant aspects of learning.

Progression depends on learners having opportunities to use higher order learning activities and develop breadth of learning through practice and application across a range of contexts. Learners will apply their skills by developing more advanced techniques as they complete questions, tasks and assessments in situations which increase in complexity. For example, a learner studying Higher mathematics will have to 'interpret the question' just as a learner in Primary 3 will, but in a different context and with increasing complexity.

Many of these skills for numeracy and mathematics have parallels in other curricular areas. In science, learners will routinely discuss their thinking and the reasonableness of their answers. In language, they will link concepts and discuss their answers to questions, justifying what they have said. Each curricular area has its own vocabulary which gives precision and aids thinking. Therefore classroom practice in developing numeracy skills can be applied in other curricular areas.

For each of the eight identified learning statements, this document gives the following:

- A reason why the development of each skill is important.
- Examples of experiences and outcomes related to identified skills.
- A key strategy for staff to use in developing the skill.
- Sample questions to illustrate how the skill can be developed in practice.

This document is designed to support all staff in planning for progression of learners' numeracy skills.

Interpret Questions

Why is it important to develop learners' skills in interpreting questions?

Learners need to be able to successfully interpret questions in order to be able to work out a solution. This involves understanding/selecting the relevant information and identifying redundant information or missing information in a question. Interpretation of a question can also include interpreting data. For example in an advert suggesting they would save money by switching mobile phone contracts, learners need to understand the question and the information presented to be able to work out the solution.

What does progression in 'Interpret Questions' look like?

I can collect objects and **ask questions** to gather information, organising and displaying my findings in different ways.

MNU 0-20a

I have **explored a variety of ways in which data is presented** and can ask and answer questions about the information it contains.

MNU 1-20a

Having **determined which calculations are needed**, I can solve problems involving whole numbers using a range of methods, sharing my approaches and solutions with others.

MNU 2-03a

I can round a number using an appropriate degree of accuracy, having **taken into account the context** of the problem.

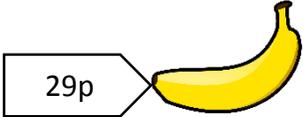
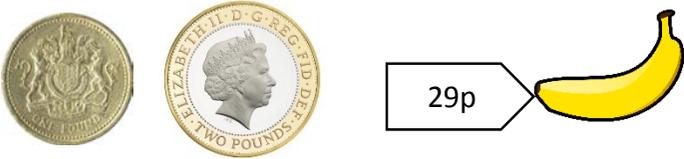
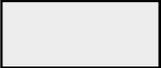
MNU 3-01a

I can **research, compare and contrast** aspects of time and time management as they impact on me.

MNU 4-10a

How can the skill of interpreting questions be developed in practice?

Limiting the amount of information given, or providing too much, will support learners in developing the ability to select the relevant information.

Question to assess knowledge	Question to develop and assess knowledge and skills
<p>Bananas cost 29p each. Tom buys 10 of them.</p> <p>How much does that cost?</p> <p>How much change does he get from £3?</p> 	<p>My friend went to the shops with £3. He likes bananas. Bananas cost 29p each.</p>  <p>How many could he buy?</p>
<p>The perimeter of this rectangle is 20m.</p>  <p>(not to scale)</p> <p>What is largest possible area of this rectangle?</p>	<p>The distance around Mr Cotter's vegetable patch measures 20m. He has bought chicken wire to put up which will keep out the rabbits because they will eat the seedlings and ruin the crop. He has decided to buy 5m rolls of chicken wire priced at £7.25.</p>  <p>What is the largest area Mr Cotter can make for his vegetables with the fencing he has purchased?</p> <p>If he digs another vegetable patch with the same perimeter but a different shape, would is the maximum area he can achieve?</p>

Select and communicate processes and solutions

Why is it important to develop learners' skills in selecting and communicating processes and solutions?

Being able to select appropriate processes depends on learners having a repertoire of strategies that they can apply after successfully interpreting the question. In real life situations there are usually several possible solutions and many ways of arriving at them. Learners need to be able to explain why they have chosen a certain process as it demonstrates their deep understanding of the task, question or assessment. Staff can discover a great deal of information about learners' understanding when they externalise their thought process, either through explaining their thinking verbally, or demonstrating it pictorially. It is said one only truly understands something when one has to explain it to someone else. This is fundamental to developing the skill of communicating processes and solutions.

What does progression in 'Select and Communicate Processes and Solutions' look like?

*I have experimented with everyday items as units of measure to investigate and compare sizes and amounts in my environment, **sharing my findings with others.***

MNU 0-11a

*I can use addition, subtraction, multiplication and division when solving problems, **making best use of the mental strategies and written skills I have developed.***

MNU 1-03a

*Having **determined which calculations are needed**, I can solve problems involving whole numbers using a range of methods, sharing my approaches and solutions with others.*

MNU 2-03a

***I can solve practical problems by applying my knowledge** of measure, choosing the appropriate units and degree of accuracy for the task and using a formula to calculate area or volume when required.*

MNU 3-11a

*I can research, compare and contrast a range of personal finance products and, after making calculations, **explain my preferred choices.***

MNU 4-09c

How can the skill of selecting and communicating processes and solutions be developed in practice?

Regularly ask learners to explain and give reasons for their strategies and solutions and when appropriate, show their working.

Question to assess knowledge	Question to develop and assess knowledge and skills
<p><i>These diagrams are the same size.</i> <i>Comparing fractions of a quantity:</i> <i>Shade $\frac{3}{4}$.</i></p>  <p><i>In this diagram shade $\frac{5}{6}$.</i></p> <p><i>Which is the larger area?</i></p> 	<p><i>Investigate:</i></p> <p><i>Would you rather have $\frac{3}{4}$ or $\frac{5}{6}$ of a bar of chocolate?</i></p> 
<p><i>A ream of paper is 7cm thick and contains 500 sheets of paper.</i></p>  <p><i>What is the thickness of each sheet?</i></p>	<p><i>How would you calculate the thickness of a sheet of paper?</i></p> 

Justify Choice of Strategy Used

Why is it important to develop learners' skills in justifying their choice of strategy used?

Justifying the choice of strategy used will allow learners to come to an understanding of why some strategies are preferred to others. For example, when learners share their strategies with others, they will not only begin to develop a wide repertoire of strategies but also identify the most efficient strategies for different types of tasks. In developing this skill in learners, staff need to consistently ask 'why' or ask learners to show their working: it is all too easy to develop an assumption that the right answer equates with a correct strategy.

What does progression in 'Justify Choice of Strategy Used' look like?

*I can use the signs and charts around me for information, helping me plan and **make choices and decisions** in my daily life.*

MNU 0-20c

*Through exploring how groups of items can be shared equally, I can find a fraction of an amount by **applying my knowledge** of division.*

MNU 1-07b

*Having discussed the variety of ways and range of media used to present data, I can interpret and draw conclusions from the information displayed, recognising that the **presentation may be misleading**.*

MNU 2-20a

*I can find the probability of a simple event happening and explain why the consequences of the event, as well as its probability, should be **considered when making choices**.*

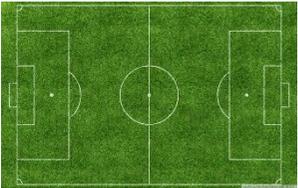
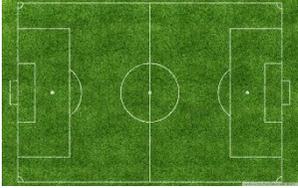
MNU 3-22a

*I can **choose the most appropriate form** of fractions, decimal fractions and percentages to use when making calculations mentally, in written form or using technology, then use my solutions to make comparisons, decisions and choices.*

MNU 4-07a

How can the skill of justifying choice of strategy used be developed in practice?

Regularly discuss different methods for calculating answers and investigate most efficient methods.

Question to assess knowledge	Question to develop and assess knowledge and skills
<p>A football pitch measures 120m by 90m.</p> <p>A  B</p> <p>How far is it to walk round the outside from A to B? How far is it to walk directly from A to B?</p>	<p>A football pitch measures 120m by 90m</p> <p>A physiotherapist runs from A to B. What is the shortest distance he could run?</p> <p>A  B</p>
<p>There are two tills in the local shop. They both have queues. It takes 10 seconds for the till operator to process an item and 30 seconds for each customer to pay.</p> <p>How long will it take for you to get to the front of the queue if there is one person in front of you with 18 items? How about a queue with five people in front of you, with 1, 3, 6, 2 and 3 items?</p>	<p>There are two tills in the local shop. They both have queues. It takes 10 seconds for the till operator to process an item and 30 seconds for each customer to pay.</p> <p>One queue has 5 people with 1, 3, 6, 2 and 3 items. The other queue has one person with 18 items.</p> <p>How do you know which is the quickest queue to join?</p>

Link Mathematical Concepts

Why is it important to develop learners' skills in linking mathematical concepts?

It is essential that when staff are planning for learning, they consider how mathematical concepts are connected and make this explicit to learners. For example, making explicit the relationship between multiplication and area in measurement makes for smooth progression in learning. Staff must ensure that they use the correct language and should avoid incorrect 'rules' such as 'multiplying makes things bigger' or 'move the decimal point' as this can cause difficulties as learners progress and concepts become more sophisticated.

What does progression in 'Link Mathematical Concepts' look like?

*I am aware of how routines and events in my world link with **times** and seasons, and have explored ways to **record and display** these using clocks, calendars and other methods.*

MNU 0-10a

*I can **estimate** how long or heavy an object is, or what amount it holds, using everyday things as a guide, then **measure** or weigh it using appropriate instruments and units.*

MNU 1-11a

*I can use and **interpret** electronic and paper-based **timetables** and schedules to plan events and activities, and make time **calculations** as part of my planning.*

MNU 2-10a

*Using simple time periods, I can work out how long a journey will take, the speed travelled at or distance covered, **using my knowledge of the link** between time, speed and distance.*

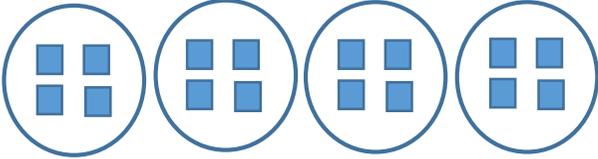
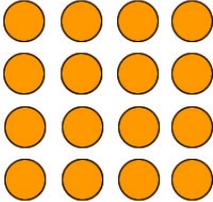
MNU 3-10a

*I can **research**, compare and contrast a range of personal finance products and, after making **calculations**, explain my preferred choices.*

MNU 4-09c

How can the skill of linking mathematical concepts be developed in practice?

Discuss and make explicit links to the different aspects of numeracy and mathematics and other curricular areas which learners are using to support their understanding of new concepts or tasks in unfamiliar contexts.

Question to assess knowledge	Question to develop and assess knowledge and skills
<p><i>Teaching multiplication:</i></p>  <p>$4 \times 4 = 16$ (<i>memorisation by rote</i>)</p>	<p><i>Use of arrays:</i></p>  <p>$4 \times \square = 16$</p> <p>$\frac{1}{4}$ of 16 =</p> <p>$4^2 =$</p> <p>$\sqrt{16} =$</p> <p><i>This four by four array visually shows that $4 \times 4 = 16$, $\frac{1}{4}$ of 16 = 4, $4^2 = 16$, $\sqrt{16} = 4$. Learners could be asked to represent numbers in this way to develop their understanding of the relational links between operations and patterns in numbers.</i></p>
<p>What is $0.4 \times 21.6\text{m}$?</p>	<p><i>There's a section of grass in the middle of Lorna's drive which is 0.4 metres by 21.6 metres.</i></p> <p><i>The contents of a bag of sand has the capacity to cover 14m^2. Is this enough sand to cover the area?</i></p>

Use Mathematical Vocabulary and Notation

Why is it important to develop learners' skills in using mathematical vocabulary and notation?

Encouraging the use of correct mathematical language provides learners with a common vocabulary and a shared understanding. Using terms such as "chimney sums" or other descriptive words to describe mathematics processes may confuse learners. Using the correct vocabulary from early years is important to ensure children have knowledge and understanding of mathematical language to build on as they progress through school. Mathematical notation is a written system used for recording concepts in mathematics. The notation uses symbols or symbolic expressions which are intended to have a precise meaning e.g. +,-. Activities which encourage learners to come to their own understanding of the benefits of using mathematical symbols and notation including their efficiency and the 'international language' they provide are a worthwhile part of progression.

What does progression in 'Use Mathematical Vocabulary and Notation' look like?

*I use practical materials and can 'count on and back' to help me to understand addition and subtraction, **recording my ideas and solutions in different ways.***

MNU 0-03a

*I can **use appropriate vocabulary to describe** the likelihood of events occurring, using the knowledge and experiences of myself and others to guide me.*

MNU 1-22a

*I can **explain** how different methods can be used to find the perimeter and area of a simple 2D shape or volume of a simple 3D object.*

MNU 2-11c

*I can use a variety of methods to solve number problems in familiar contexts, **clearly communicating** my processes and solutions.*

MNU 3-03a

*I can evaluate and interpret raw and graphical data using a variety of methods, comment on relationships I observe within the data and **communicate my findings** to others.*

MNU 4-20a

How can the skill of using mathematical vocabulary and notation be developed in practice?

Ensure learners are regularly articulating understanding, both verbally and in written formats, using the correct mathematical terminology and notation.

Question to assess knowledge	Question to develop and assess knowledge and skills
<p data-bbox="188 427 853 464"><i>What are the perimeter and area of this rectangle?</i></p> 	<p data-bbox="1149 427 1700 464"><i>I have a space for a rectangular enclosure.</i></p> <p data-bbox="1149 507 1648 544"><i>I am building an enclosure for my pigs.</i></p> <p data-bbox="1149 587 2018 624"><i>I will build it with 18 fence panels, each measuring 1.5 metres long.</i></p> <p data-bbox="1149 667 1693 703"><i>What is the largest area of my enclosure?</i></p>
<p data-bbox="188 782 495 818"><i>Calculate the following:</i></p> $\begin{array}{r} 28 \\ \times 7 \\ \hline \end{array}$	<p data-bbox="1149 782 1980 818"><i>There are 28 pupils in a class and each pupil was given 7 sweets.</i></p> <p data-bbox="1149 861 2029 898"><i>Explain how to find the total number of sweets needed for the class.</i></p>

Mental Agility

Why is it important to develop learners' skills in mental agility?

Mental agility is an essential skill. As adults, we need to estimate and calculate every day. This skill must be developed from the early stages. In all learning in numeracy and mathematics, lack of fluency in mental processes is a major barrier to progress. Mental agility is not how fast learners can do calculations although this is important. Mental agility is the ability to carry out multi-step mental questions accurately, either without writing anything down or without a calculator.

What does progression in 'Mental Agility' look like?

*I have explored numbers, understanding that they represent quantities, and I **can use them** to count, create sequences and describe order.*

MNU 0-02a

*I can share ideas with others to develop ways of **estimating the answer to a calculation** or problem, work out the actual answer, then check my solution by comparing it with the estimate.*

MNU 1-01a

*I can show the equivalent forms of simple fractions, decimal fractions and percentages and **can choose my preferred form** when solving a problem, explaining my choice of method.*

MNU 2-07b

*I can continue to **recall number facts quickly** and use them accurately when making calculations.*

MNU 3-03b

*Having **recognised similarities between new problems and problems I have solved before**, I can carry out the necessary calculations to solve problems set in unfamiliar contexts.*

MNU 4-03a

How can the skill of mental agility be developed in practice?

Developing efficient mental strategies is a useful life skill when learners need to calculate without the aid of a calculator.

Question to assess knowledge	Question to develop and assess knowledge and skills
<p data-bbox="439 421 819 528"><i>What percentage is 84 of 48? Or 84 is what percentage of 48?</i></p>	<p data-bbox="1151 379 1951 451"><i>A farmer successfully hatched 84 eggs laid by his 48 chickens. What percentage of chicks did he yield from his chickens?</i></p> 
<p data-bbox="188 735 913 962"><i>A boy bought crisps and was given 13 pence change. He then bought sweets and was given 18 pence change. How much change was he given altogether?</i></p> 	<p data-bbox="1093 735 1951 882"><i>A boy went into a shop and bought crisps which cost 14 pence and sweets which cost 19 pence. How much change did he get from his pound coin?</i></p> 

Algebraic Reasoning

Why is it important to develop learners' skills in algebraic reasoning?

Understanding that numbers can be replaced by pictures/symbols is fundamental to all algebraic reasoning. Using pictures and symbols is an early step in understanding variables and constants and introduces the concept of finding the unknown quantity.

This takes learners beyond the immediate, single numerical problem to working with similar problems which can be solved algebraically.

What does progression in 'Algebraic Reasoning' look like?

I can **match objects, and sort** using my own and others' criteria, sharing my ideas with others.

MNU 0-20b

Having explored fractions by taking part in practical activities, I can **show my understanding of:**

- how a single item can be shared equally
- the **notation** and vocabulary associated with fractions
- where simple fractions lie on the number line.

MNU 1-07a

Having determined which calculations are needed, I can solve problems involving whole numbers using a **range of methods**, sharing my approaches and solutions with others.

MNU 2-03a

I can **solve practical problems** by applying my knowledge of measure, choosing the appropriate units and degree of accuracy for the task and **using a formula** to calculate area or volume when required.

MNU 3-11a

I can use the link between time, speed and distance to carry out related calculations.

MNU 4-10b

How can the skill of algebraic reasoning be developed in practice?

Look for opportunities to generalise answers and develop rules rather than solving one-off questions. This makes learning transferrable.

Question to assess knowledge	Question to develop and assess knowledge and skills
<p><i>What comes next in the arithmetic sequence 3, 5, 7, 9, ... ?</i></p>	<p><i>What is the rule for finding the next term of the arithmetic sequence 3, 5, 7, 9?</i></p> <p><i>Can you work out how to calculate the fiftieth term (without writing them all down!)?</i></p>
<p><i>An antique necklace is bought for £100. After six months it has increased in value by 10%. A year later this new value has dropped by 5%.</i></p> <p><i>How much is the necklace now worth?</i></p>	<p><i>Six months after it is purchased an antique necklace has increased in value by 10%. A year later the new value of the necklace has gone down by 5%.</i></p> <p><i>Does this mean that it is now worth 95% of its original purchase price? Explain your thinking.</i></p>

Determine the Reasonableness of a Solution

Why is it important to develop learners' progress in determining the reasonableness of a solution?

As calculations in real-life and relevant contexts become more and more complex, it is likely that learners will make estimates based on the data, have to choose an appropriate strategy, and use a calculator or computers to carry out the actual calculations. At all stages, there is the possibility of errors; errors in inputting data or errors in not choosing the most efficient method. It is essential that estimates are revisited to support learners in judging how realistic their answers are. This skill needs to develop from an early age where learners consider the context of the problem in deciding whether their answer is sensible and realistic. From the earliest stages, or where calculations are simple and abstract (e.g. $5 + 6$), learners should be prompted look at the solution and try to decide whether it is reasonable or not.

What does progression in 'Determine the Reasonableness of a Solution' look like?

*I am developing a sense of size and amount by observing, exploring, using and communicating with others about **things in the world around me.***

MNU 0-01a

*I can share ideas with others to develop ways of estimating the answer to a calculation or problem, work out the actual answer, then check my solution by **comparing it with the estimate.***

MNU 1-01a

*I can use my knowledge of rounding to **routinely estimate** the answer to a problem then, after calculating, **decide if my answer is reasonable,** sharing my solution with others.*

MNU 2-01a

*I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, **using my answers to make comparisons and informed choices for real-life situations.***

MNU 3-07a

*By applying my understanding of probability, I can determine how many times I expect an event to occur, and use this information to **make predictions, risk assessment, informed choices and decisions.***

MNU 4-22a

How can the skill of determining the reasonableness of a solution be developed in practice?

Always estimate in order to efficiently interrogate answers to determine whether they are reasonable, especially in real-life situations.

Question to assess knowledge	Question to develop and assess knowledge and skills
<p><i>Usain Bolt ran 100 metres in 9.58 seconds in the 2012 Olympics. Using the Speed distance time formula, work out the speed at which Usain Bolt ran the 100m sprint.</i></p>	<p><i>Usain Bolt ran 100 metres in 9.58 seconds in the 2012 Olympics. My friend worked out that Usain could run a kilometre in under 95.8 seconds. Do you think that's right? Why?</i></p> 
<p><i>There are 25 pupils in a class and each one is given a new pencil.</i></p> <p><i>Pencils are bought in packs of six.</i></p> <p><i>Five packs of pencils are bought.</i></p> <p><i>How many pencils are left over?</i></p> 	<p><i>There are 25 pupils in a class and each one is given a new pencil.</i></p> <p><i>Pencils are bought in packs of six.</i></p> <p><i>How many packs of pencils will need to be bought?</i></p> 