Education Scotland
Foghlam Alba

Fractions, Decimal Fractions and Percentages

Professional Learning Resource

This resource is part of the suite of Numeracy Professional Learning Resources.

For Scotland's learners, with Scotland's educators
Do luchd-ionnsachaidh na h-Alba, le luchd-foghlaim Alba

Contents

Introduction

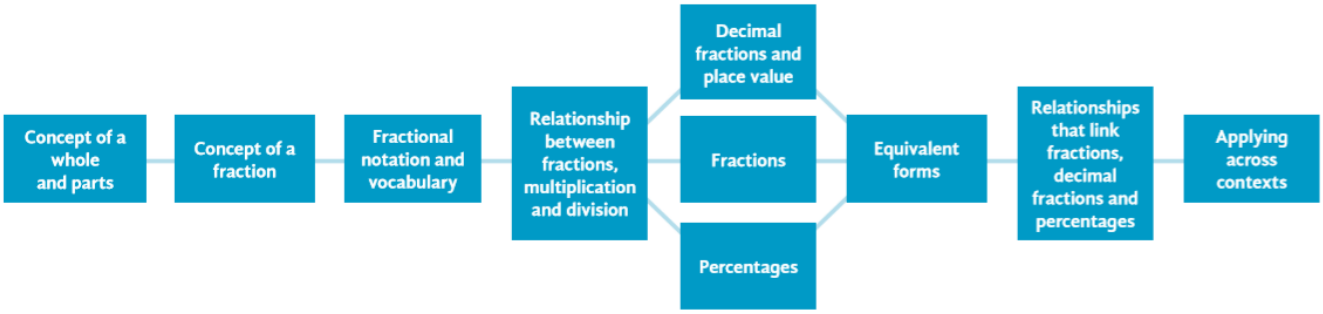
This professional learning resource has been created to enable practitioners to reflect on their own knowledge and understanding, highlight areas which children find challenging and outline effective approaches to support future learning and teaching in fractions, decimal fractions and percentages.

Fractions, decimal fractions and percentages are integral to a number of areas of numeracy and mathematics, in particular when working with money, measurement and data. Learners' understanding can be applied through a wide variety of everyday contexts and other areas of the curriculum.

Although this document's main purpose is to focus on numeracy experiences and outcomes and benchmarks, the mathematics benchmarks associated with fractions have been included as the numeracy and mathematics skills are closely interlinked in this area.

The addition, subtraction, multiplication and division of decimal fractions are discussed in the Number and Number Processes professional learning resource.

National Numeracy and Mathematics Progression Framework[1]
Fractions, Decimal Fractions and Percentages



[1] Further guidance on using the National Numeracy and Mathematics Progression Framework can be found by clicking here.

Early Level

The table below includes the experiences and outcomes and benchmarks related to 'Fractions, Decimal Fractions and Percentages' at early level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	Benchmarks
I can share out a group of items by making smaller groups and can split a whole object into smaller parts. **MNU 0-07a**	• Splits a whole into smaller parts and explains that equal parts are the same size. • Uses appropriate vocabulary to describe halves. • Shares out a group of items equally into smaller groups.

Effective Learning and Teaching Approaches

Careful consideration should be given to the spaces, interactions and experiences we provide, ensuring that opportunities are provided for learners to develop the concepts of sharing, grouping and equal parts.

Spaces, Interactions and Experiences

Rich, carefully considered learning spaces both outdoor and indoor can offer learners practical opportunities to develop their concept of sharing, grouping and equal parts. The choice of experiences on offer should reflect an environment of open-ended possibilities in which children can feel intrinsically motivated to explore and investigate through play. Selecting appropriate and engaging resources can enhance interactions, leading to creativity and curiosity.

Open-ended materials offer the potential for creative explorations through child-initiated and adult initiated learning experiences. Spaces should be planned to provide a balance of opportunities for learners to play, explore, investigate and question. Practitioners should ensure that planning for learning is carefully balanced and is both responsive and intentional in design. These opportunities should enable learners to make sense of sharing, grouping and equal parts in the world around them, whilst also ensuring learners needs are being met through their engagement with all experiences and outcomes presented within early level.

Please see the 'Guidance Document' for additional information for learning and teaching approaches at early level.

Fractions can be an abstract concept for learners, and they should be offered lots of practical and real life examples involving fractions. Early experience of fractions should involve halves of physical objects such as fruit or malleable dough. Learners may share items such as a biscuit or a banana with a friend so that they get half each. They may fold paper shapes to create halves. Through such early experiences, learners begin to understand that fractions are created when a whole object is divided into equal parts, allowing them to 'see' the whole and the associated parts.

A wide variety of concrete materials can be used both indoors and outdoors to develop understanding and skills in sharing, grouping and equal parts. Some examples of possible objects that can support this are listed below, however materials provided can be adapted to reflect learners' interests and ideas.

Provide a wide variety of different materials and objects.

buttons	marbles/beads	feathers	flowers	leaves
sticks	shapes	socks	money	stones
tubs/hoops/ containers	toy cars	toy animals	toy/real food	coins

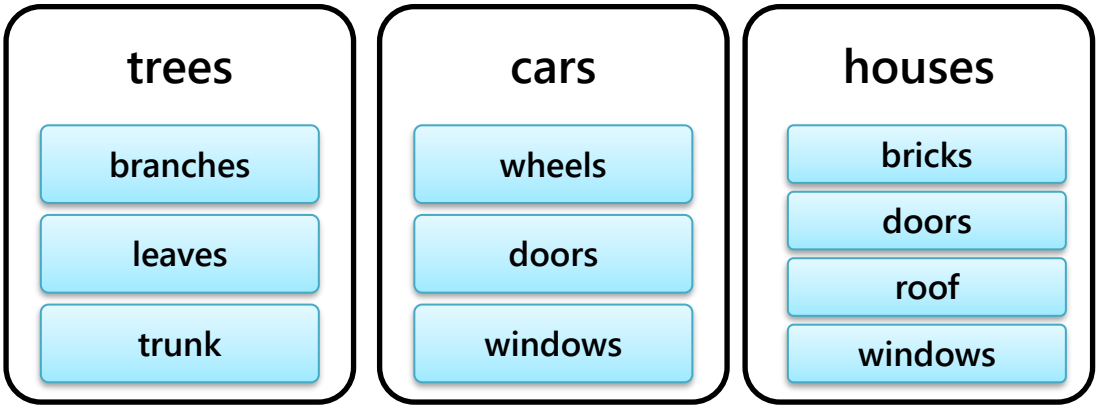
Practitioners should provide opportunities for learners to experience taking whole things apart and putting them back together. This will develop an understanding that many objects and items around them are made up of different parts and that these different parts make a whole. Some examples of these types of items are noted below.

small world play objects

construction kits

Old household objects to tinker with, e.g. broken clocks, old electrical items etc.

Objects in the wider environment can also be used, allowing learners to see the whole and the associated parts.



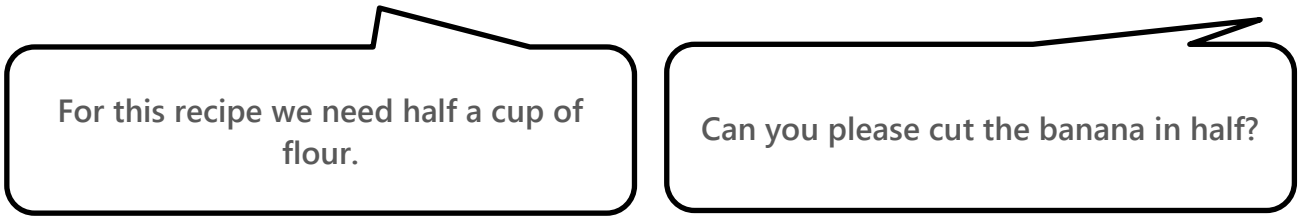
I wonder what the different parts of that (car) are?

These are some of the suggestions that learners may have. It is likely they will have other ideas too.

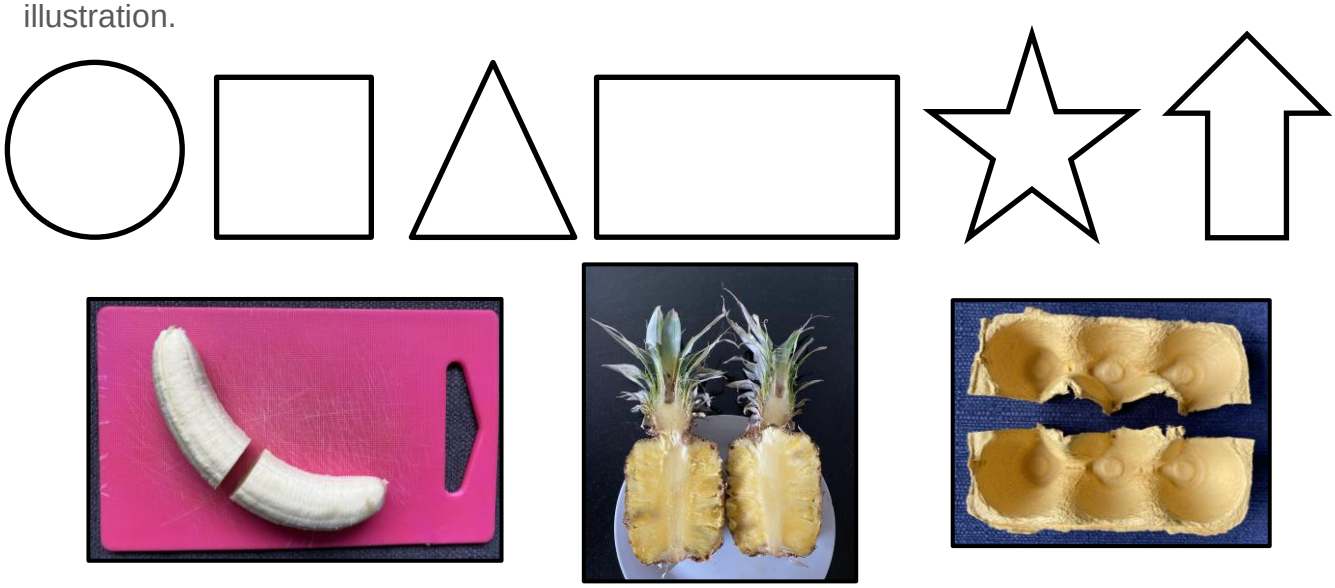
Halving and Doubling

Cutting shapes and soft materials like malleable dough or fruit can allow children to investigate equal and unequal parts of different shapes and objects. It is important that children know and can explain that a shape or object is halved by splitting it into two **equal** sized pieces. Paper folding and cutting activities can also reinforce the idea of halves. It can also be beneficial to encourage learners to put two halves back together to reinforce that they make 'a whole'. It is important to note that when an object is split, if the resulting parts are unequal then each part cannot be a half. This should be made clear through practical demonstrations and dialogue with learners.

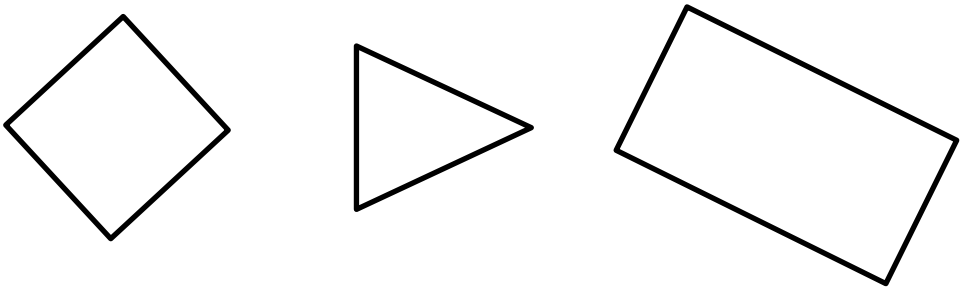
Everyday experiences such as snack time and baking can support the development of halves and doubles.



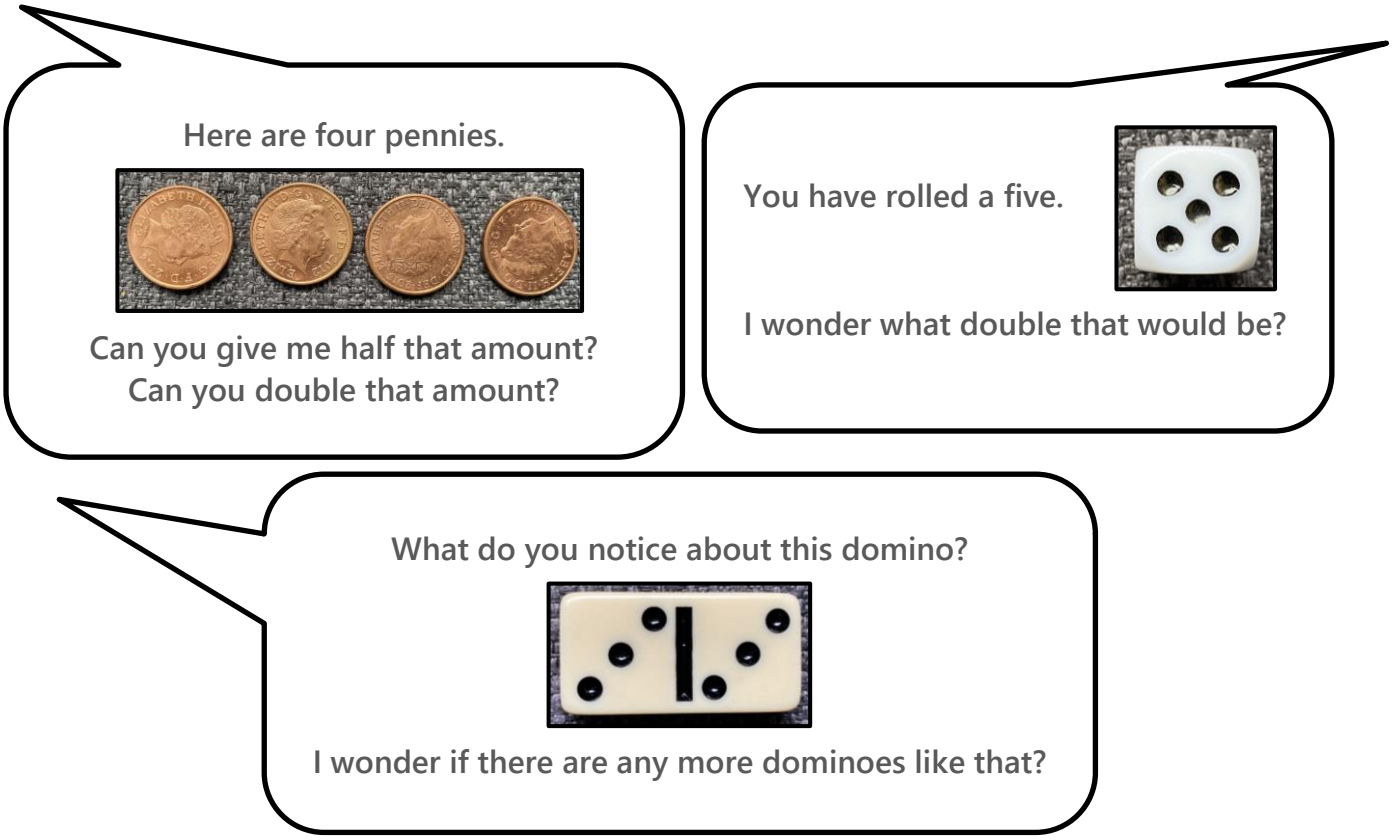
It is important to note that learners should experience a range of different shapes and objects being partitioned equally. Pizzas and cakes are typically used as examples but learners need experience of partitioning a variety of shapes and objects. Some examples are included for illustration.



It is also beneficial to partition regular shapes equally when they are laid out different ways.



Halving and doubling can be reinforced using coins, dice and dominoes.



When investigating measure, the descriptive language of ‘full’ and ‘empty’ could be extended to use of the term ‘half full’. Halving and doubling in the context of length can also be explored.

The concept of halfway can be introduced in a variety of situations, for example halfway along a netball court, halfway along a piece of ribbon and halfway up a door.

Grouping and Sharing

Learners should be provided with opportunities to share out items and investigate the concepts of ‘sharing equally’ and ‘grouping’.

Through grouping and sharing small quantities, learners begin to lay the foundations for multiplication and division, doubling numbers and quantities. They can make connections between arrays and number patterns.

In equal sharing learners would start with the number of groups and share the objects equally between each group.

I wonder what the best way to share these strawberries between you and your friend is?

There are three of us here today and we have nine sticks. How can we make sure we all have the same number of sticks?

In grouping, learners need to know the number of objects each group should receive. They also know what the total number of objects to begin with is.

We have ten leaves. I wonder if we can make groups of two with them?

There are nine of us here today. Can we get into groups of three?

At early level, practitioners are encouraged to provide learners with a range of opportunities to share out a number of items into smaller, equal groupings. This can be in a variety of contexts, for example sharing out pieces of fruit at snack time, sharing craft materials, creating equal piles of beanbags in PE games or hunting for and sharing out pine cones during outdoor learning activities.

Learners can be encouraged to collect items of their choice. In these activities, natural links can be made to early multiplication and division skills and the distinction should be made between sharing and grouping.

It is important for learners to be able to recognise equal **and** unequal shares of concrete materials and to be able to explain why shares are equal or unequal.

Books which have implicit or explicit links to mathematical concepts are an excellent resource to use when teaching fractions. The use of storytelling as a learning vehicle can promote excitement about fractions, deepen understanding of fractional concepts, and add an element of fun and creativity. The following list of suggestions will hopefully capture learners' attention and promote engagement.

Books to consider:

- The Doorbell Rang by Pat Hutchins
- Equal Shmequal by Virginia Kroll
- A Fair Bear Share by Stuart J Murphy
- One Hundred Angry Ants By Elinor J Pinczes
- One Thing by Lauren Child
- Each Orange had 8 Slices By Paul Giganti

Links to Other Curriculum Organisers

Number and number processes

- Doubles numbers to a total of ten.
- Partitions quantities of ten into two or more parts.
- Grouping items.

Money

- Doubling, halving, sharing and grouping within ten.

Measurement

- Filling half of a container, finding half way between two points.

Properties of 2D shape and 3D objects

- Splitting a variety of shapes and objects in half.

Symmetry

- Creating symmetrical pictures with one line of symmetry.

Food technology

- Preparing and presenting food.

Reflective Questions

- **What opportunities can we use, throughout the day, to practise and reinforce sharing, grouping, equal parts, halving and doubling?**
- **How do we make clear the concept of halves and non-halves using concrete materials and progressing to pictorial representations?**
- **How do we make clear the distinction between equal and non-equal shares with concrete materials and progressing to pictorial representations?**
- **How do we ensure all practitioners are using the correct mathematical vocabulary?**

First Level

The table below includes the experiences and outcomes and benchmarks related to 'Fractions, Decimal Fractions and Percentages' at first level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	Benchmarks
Having explored fractions by taking part in practical activities, I can show my understanding of: • how a single item can be shared equally; • the notation and vocabulary associated with fractions; and • where simple fractions lie on the number line. **MNU 1-07a** Through exploring how groups of items can be shared equally, I can find a fraction of an amount by applying my knowledge of division. **MNU 1-07b** *Through taking part in practical activities including use of pictorial representations, I can demonstrate my understanding of simple fractions which are equivalent.* ***MTH 1-07c*** I can use addition, subtraction, multiplication and division when solving problems, making best use of the mental strategies and written skills I have developed. **MNU 1-03a**	• Explains what a fraction is using concrete materials, pictorial representations and appropriate mathematical vocabulary. • Demonstrates understanding that the greater the number of equal parts, the smaller the size of each share. • Uses the correct notation for common fractions to tenths, for example, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{5}{8}$. • Compares the size of fractions and places simple fractions in order on a number line. • Uses pictorial representations and other models to demonstrate understanding of simple equivalent fractions, for example , $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$. • Explains the role of the numerator and denominator. • Uses known multiplication and division facts and other strategies to find unit fractions of whole numbers, for example, $\frac{1}{2}$ or $\frac{1}{4}$. • Uses multiplication and division facts to solve problems within the number range 0 to 1000.

Effective Learning and Teaching Approaches

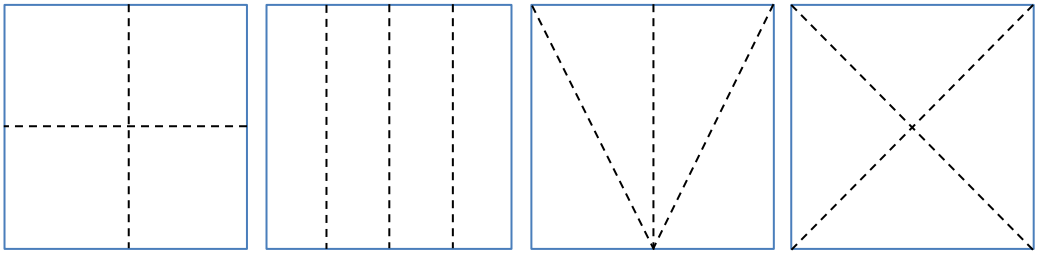
It is important that learners develop a connected and flexible sense of fractions at this level. The use of concrete materials and pictorial representations can assist learners in deepening their understanding. Examples of these can be found on the next page.

Fold paper in halves, quarters, eighths, etc.

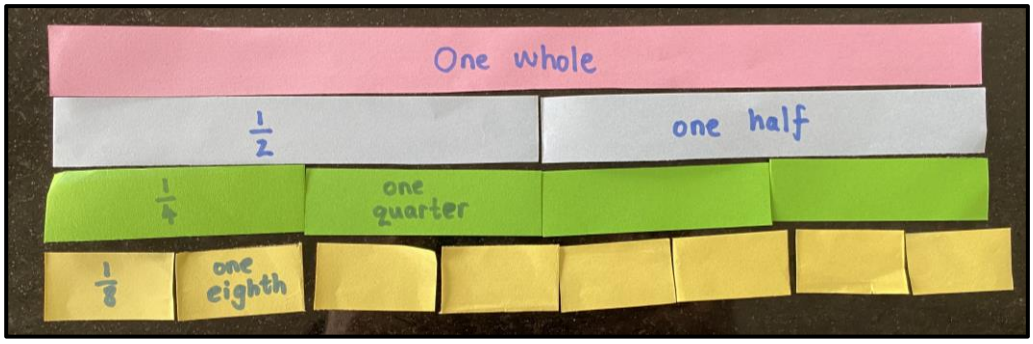
Discussion about what happens each time we halve the paper.

Fold paper plates in halves, quarters, eighths, etc.

Learners can explore different representations of halves and quarters through paper folding activities. They can be asked to fold simple shapes such as squares and rectangles in half, and half again, in as many different ways as they can. When unfolded, the folds can be used as the edges of equal sections. Learners could be challenged to find all the possible ways to use two folds to split a square into quarters.

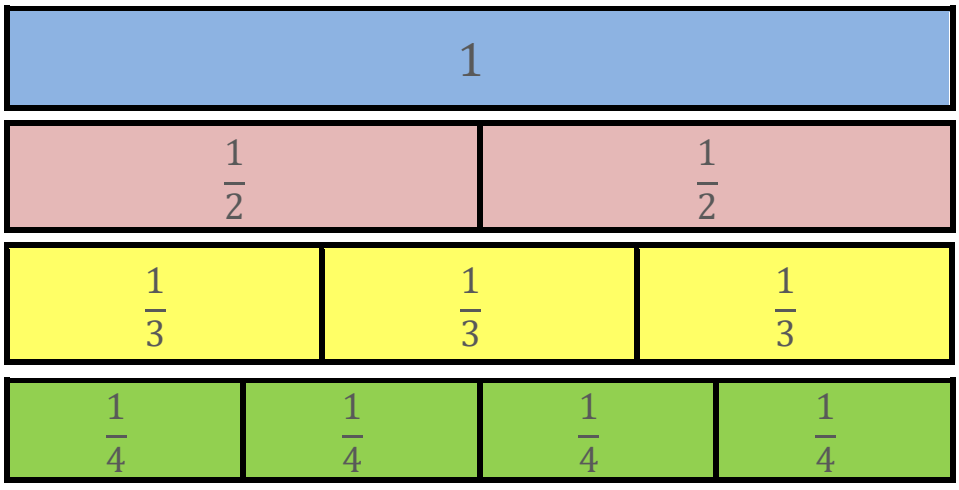


This activity could be extended, for example to making eighths with three folds. Learners could also explore what happens if they are allowed to use more than two or three folds to make quarters or eighths. Activities like this will help learners develop a flexible view of what quarters of a shape can look like. Folding activities like this can also be used to illustrate clearly that the greater the number of equal parts, the smaller the size of each part.



Create fraction strips and use fraction walls to make comparisons. *'How many different ways can you make one half?'*

Digital versions are available and can also be effective resources to support this process. These manipulatives can illustrate the relative sizes of different fractions, as well as their equivalence.



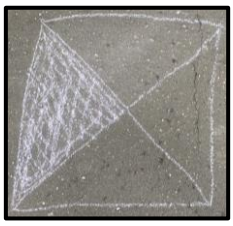
Everyday items such as interlocking building blocks can also be used creatively to represent fractions and equivalent fractions.

Collect materials and use them to calculate fractions.

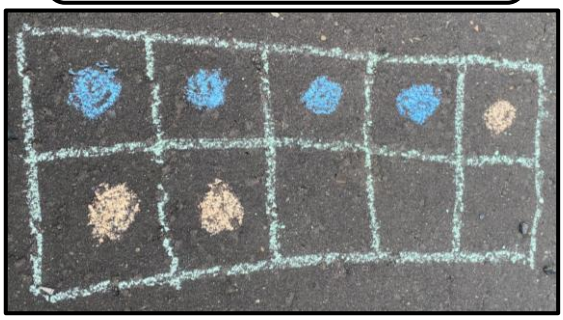
'*I have collected four items. Three quarters of these items are leaves.*'

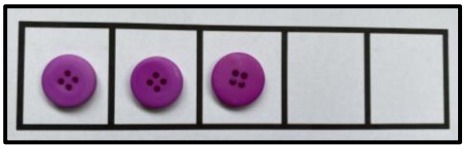
Learners could draw shapes in the playground and shade in fractions of the shapes.

It is important to encourage learners to draw a variety of shapes.



Using five and ten frames.





$\frac{3}{5}$ of the squares have buttons in them.

$\frac{2}{5}$ of the squares have nothing in them.

$\frac{4}{10}$ of the squares have blue dots in them.

$\frac{3}{10}$ of the squares have orange dots in them.

$\frac{3}{10}$ of the squares have no dots in them.

$\frac{5}{10}$ of the squares have leaves in them.

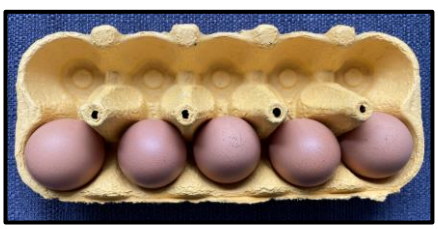
$\frac{5}{10}$ of the squares have nothing in them.

Some learners may realise that this is the same as one half. Equivalence will be explored further in second level.

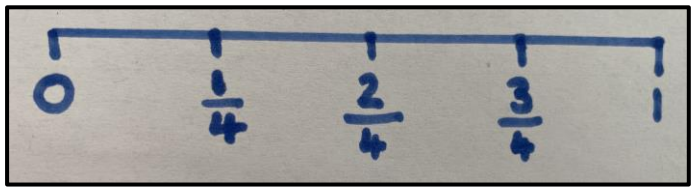
Use containers and real life objects to investigate fractions.

'This bottle is approximately half full'
'Half of the eggs are in the box.'

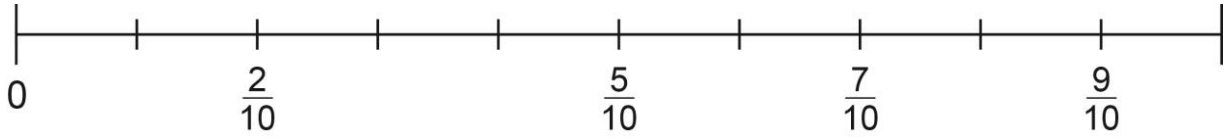
This can be extended to quarters, thirds etc.



Create fraction number lines.
In this example the number line has been created using quarters, learners can also create them using thirds, fifths, tenths etc.



Learners can also be asked to complete number lines with missing fractions. These can also be used to explore counting sequences and to develop an understanding of the relative sizes of fractions with a common denominator.



Use outdoor objects to arrange materials and create linking facts.

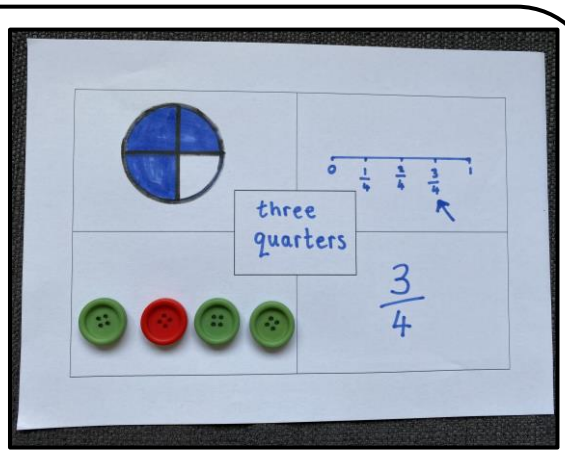
$\frac{1}{4}$ of 20 is 5 $\frac{2}{4}$ of 20 is 10 $\frac{3}{4}$ of 20 is 15 $\frac{4}{4}$ of 20 is 20

Clear links should be made between fractions, multiplication and division as this will benefit learners when moving to abstract calculations.

4 x 5 = 20 5 x 4 = 20 $\frac{1}{4}$ of 20 is 5

20 ÷ 4 = 5 20 ÷5 = 4 $\frac{1}{5}$ of 20 is 4

Show fractions represented in a variety of ways.



Explore lots of examples of shapes being shared in equal parts to show that it can be done in a number of different ways.

Provide lots of opportunities to find fractions of shapes and objects such as fruit, food, malleable dough, etc.

Explore the meaning of the word fraction. Discuss where learners have encountered fractions in their lives.

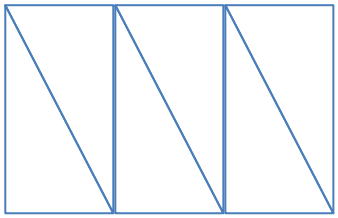
Fraction of a whole

Learners can further develop their understanding of a fraction of a whole through pictorial representations, for example by shading sections within a diagram of a 2-dimensional shape with equal sections. A simple example is given below, where learners can develop an understanding of the role of numerator and denominator using the language “one part out of three parts”.

Shade one third $\left(\frac{1}{3}\right)$

It is important that learners have opportunities to explore a variety of ways that equivalent fractions can be represented.

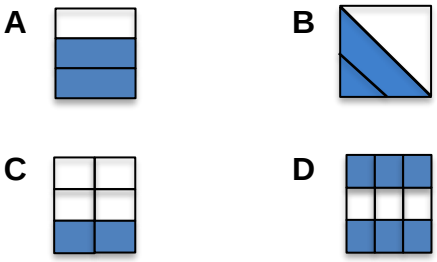
Shade one third $\left(\frac{1}{3}\right)$ in as many different ways as you can.



When learners are becoming confident with shading parts of a simple diagram to represent a fraction of a whole, it is important that they have the opportunity to use a variety of shapes. For example, the diagrams below show some less familiar ways to shade halves and illustrate that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$.

Exposure to examples and non-examples is also important to identify and address misconceptions. In the example below, learners who have not fully understood the importance of equal sized sections may circle A and B instead of A and D.

Two of the diagrams show $\frac{2}{3}$ shaded. Circle both.



Fraction of a quantity

Finding a fraction of a quantity should form an explicit part of learners' understanding and experience of multiplication and division. They need to develop an understanding of the meaning of mathematical terminology and notation associated with fractions, particularly the meaning of numerator and denominator. They should have the opportunity to solve a wide range of multiplication and division problems with an emphasis on mental strategies. It is important that the concepts of grouping and sharing are reinforced, along with how this knowledge links to finding a fraction of a quantity.

This link can be made explicit by forming families of facts such as the example given below.

3 x 2 = 6 2 x 3 = 6 6 ÷ 2 = 3 6 ÷ 3 = 2 $\frac{1}{2}$ of 6 = 3 $\frac{1}{3}$ of 6 = 2

This can lead on to the first steps in algebraic thinking:

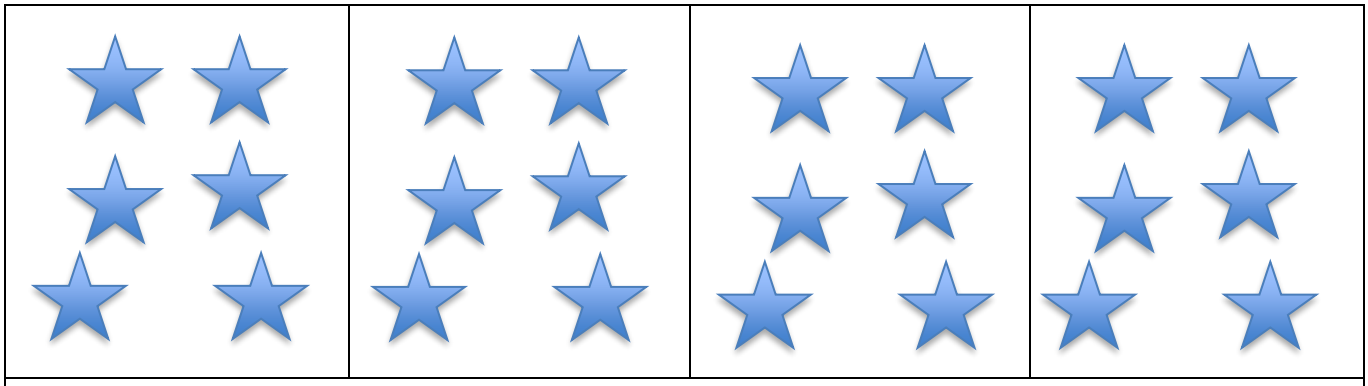
12 ÷ ☐ = 3?

$\frac{1}{3}$ of ☐ = 10?

☐ x 4 = 32?

Using concrete materials and pictorial representations can help learners apply their experience of sharing and division to find a simple fraction of a quantity as shown in the example below. Learners could share out items such as counters, cubes, buttons etc, before moving on to these pictorial representations.

Find one quarter of 24 stars



$\frac{1}{4}$ of 24
= 24 ÷ 4
= 6

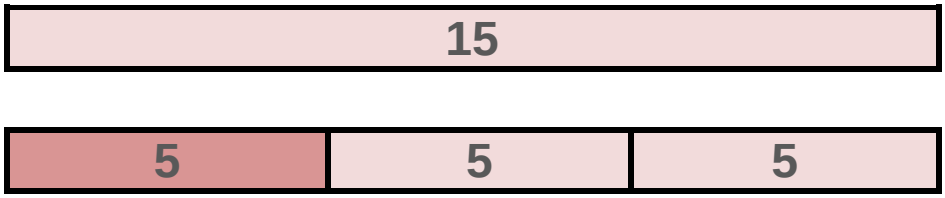
It is important that learners are given the opportunity to become familiar with the notation of fractions, and to link it to their understanding of fractions of a whole, to equal shares and hence to division. This will help enable them to find a fraction of an amount when the fraction is expressed in mathematical notation within a simple word problem, such as the example given below.

A baker drops a box of 15 eggs.

$\frac{1}{3}$ of the eggs break

How many of the eggs break?

It is helpful to allow learners to use concrete materials or pictorial representations to work out their answers until they are confident in their understanding. Bar modelling can be particularly helpful for the example above.



When learners are confident in their understanding, they can begin to tackle problems that move beyond known facts, such as the example given below.

84 pupils are taking part in the school's sports day.

$\frac{1}{6}$ of them are competing in the long jump.

How many pupils are competing in the long jump?

14 pupils

When solving such problems, learners will need to make use of strategies they have developed for the division of larger numbers. For more information on these strategies, please refer to the professional learning resource on Number and Number Processes.

Points to consider

- Fractions should be recorded correctly, for example $\frac{3}{4}$ and not ¾ .
- Careful and considered language is important, for example $\frac{1}{3}$ should be referred to as 'one third' rather than 'a third' or 'one over three'.
- Some learners may need further support with understanding that the larger the denominator the smaller the share. Learners could create support cards that they can access to remind them what the difference between the numerator and denominator is.

Links to Other Curriculum Organisers

The use of fractions has natural and explicit applications within other parts of the numeracy and mathematics curriculum. This repeated exposure deepens learning and, when highlighted explicitly, can enable learners to see how different aspects of numeracy and mathematics are interconnected. Fractions are mentioned explicitly in the following benchmarks.

Time

- Tells the time using half past, quarter past and quarter to using analogue and digital 12 hour clocks.

Measurement:

- Reads a variety of scales on measuring devices including those with simple fractions, for example, $\frac{1}{2}$ litre.
- Uses square grids to estimate then measure the areas of a variety of simple 2D shapes to the nearest half square.
- Creates shapes with a given area to the nearest half square using square tiles or grids.

Angle

- Uses technology and other methods to describe, follow and record directions using words associated with angles, directions and turns including, full turn, half turn, quarter turn, clockwise, anticlockwise, right turn, left turn and right angle.

Work on fractions (for example fractions of a quantity) can also be reinforced when solving problems in other curricular organisers such as Money, Data and Analysis, Ideas of Chance and Uncertainty, and in other areas of the wider curriculum.

Reflective Questions

- **In our interactions with children, how can we describe fractions as ‘parts of a whole’ other than relating to slices of cake or pizza?**
- **How can we use concrete materials to show the link between division and fractions?**
- **How often do we discuss non examples with learners in order to further develop their understanding?**
- **How can outdoor learning enrich the teaching of fractions, decimal fractions and percentages?**
- **Which concrete materials do we use to develop understanding of equivalence?**

Second Level

The table below includes the experiences and outcomes and benchmarks related to 'Fractions, Decimal Fractions and Percentages' at second level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	Benchmarks
I have investigated the everyday contexts in which simple fractions, percentages or decimal fractions are used and can carry out the necessary calculations to solve related problems. **MNU 2-07a** I can show the equivalent forms of simple fractions, decimal fractions and percentages, and can choose my preferred form when solving a problem, explaining my choice of method. **MNU 2-07b** *I have investigated how a set of equivalent fractions can be created, understanding the meaning of simplest form, and can apply my knowledge to compare and order the most commonly used fractions.* ***MTH 2-07c***	• Uses knowledge of equivalent forms of common fractions, decimal fractions and percentages, for example, $\frac{3}{4}$ = 0.75 = 75%, to solve problems. • Calculates simple percentages of a quantity, and uses this knowledge to solve problems in everyday contexts, for example, calculates the sale price of an item with a discount of 15%. • Calculates simple fractions of a quantity and uses this knowledge to solve problems, for example, find $\frac{3}{5}$ of 60. • Creates equivalent fractions and uses this knowledge to put a set of most commonly used fractions in order. • Expresses fractions in their simplest form.

Effective Learning and Teaching Approaches

Simple problems involving money and metric measurement can be a useful context to explore fractions, decimal fractions and percentages at this level. Examples of these can be found within the Number and Number processes professional learning resource.

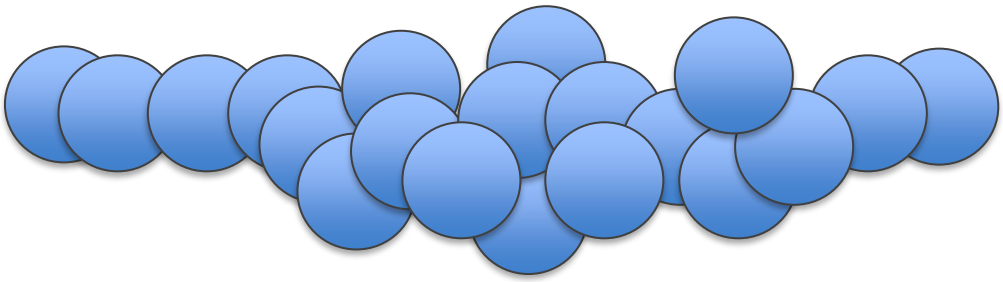
It is important that learners are provided with opportunities to explore alternative strategies and discuss these with their peers. Activities which encourage learners to talk about number provide them with the opportunity to identify a wide range of possible approaches to a solution and develop reasoning skills and competency. Examples of these are included throughout this level.

Fraction of a Quantity

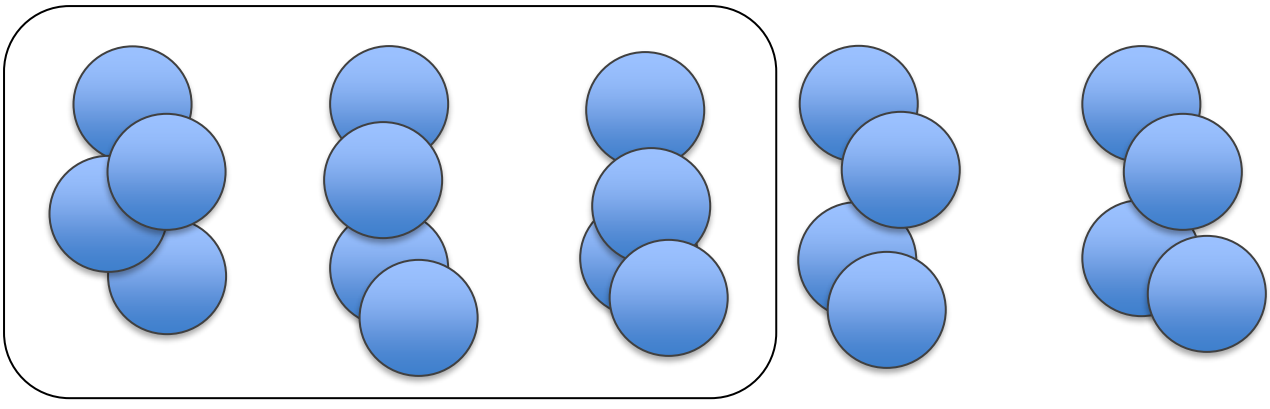
The mathematical language and notation of fractions must be used correctly and often, as it is vital that learners are clear about the role of the numerator and the denominator when finding non-unitary fractions of quantities in problems such as the one below.

> The vet treated 120 animals last week.
>
> $\frac{9}{10}$ of them are were rabbits.
>
> How many rabbits did the vet treat last week?
>
> **108 rabbits**

Concrete materials or pictorial representations can be used to reinforce understanding of the use of division (where the denominator gives the number of shares to be made) and multiplication (where the numerator gives the number of shares to be included) until learners become confident in tackling such questions.

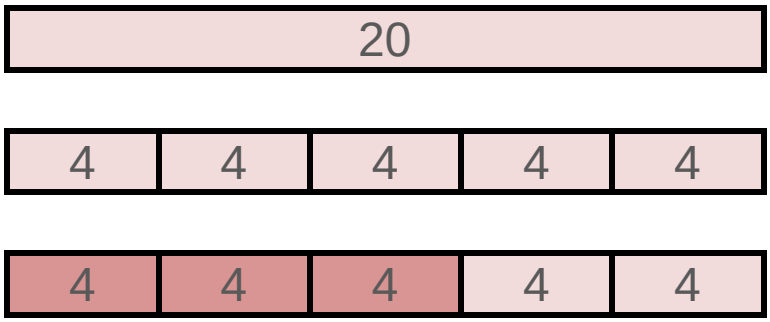
> **Here are 20 counters. Sue picks up $\frac{3}{5}$ of them.**
>
> **How many counters does Sue pick up?**
>
> 

Learners could use counters or dots on a page to work out their solution:



12 counters

Alternatively, bar modelling could be used to help reach the same answer:

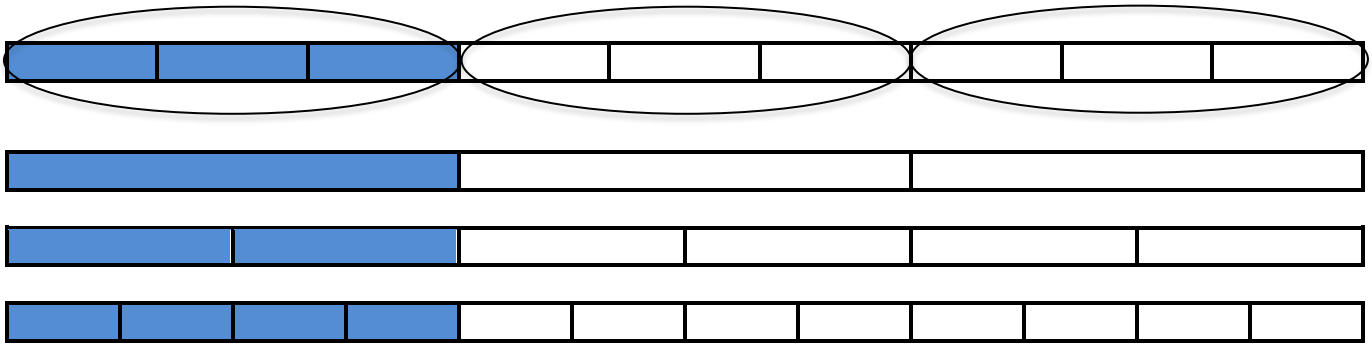


Equivalent Fractions

The use of concrete materials and pictorial representations can assist learners in strengthening their understanding of equivalent fractions and simplifying fractions. It is vital that learners do not come to view finding equivalent fractions and simplifying fractions as separate skills, and that they are taught and practised in a connected way, for example:

$$\frac{\mathbf{3}}{\mathbf{9}} = \frac{?}{\mathbf{3}} = \frac{\mathbf{2}}{?} = \frac{\mathbf{4}}{?}$$

A pictorial approach could be used to complete this task, as shown below:



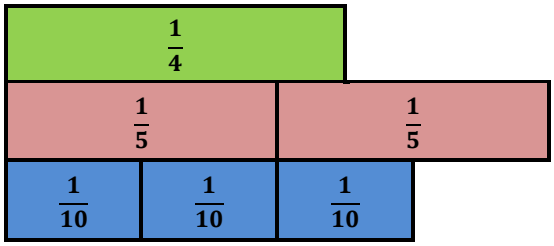
Ordering Fractions

Tasks involving ordering fractions with different denominators provide a context in which learners can apply their knowledge and understanding of equivalent fractions and common multiples, for example:

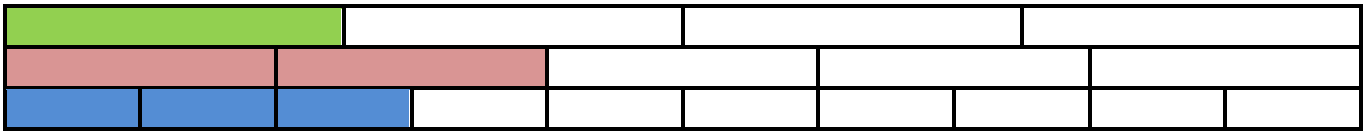
Put these fractions in order of size, starting with the smallest.

$\frac{2}{5}$ $\frac{3}{10}$ $\frac{1}{4}$

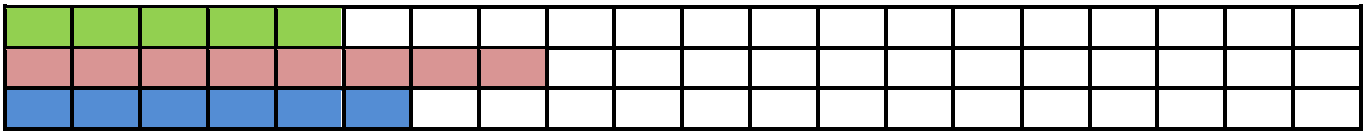
Manipulatives (fraction bars) could be used to illustrate the relative size of each of the fractions as shown below:



A pictorial approach could also be used.



In this case, careful consideration of the length of the bar model would need to be made, for example by using 10 cm lengths. This approach could then lead on to using the least common multiple of 4, 5 and 10 to convert each fraction to twentieths.



$\frac{5}{20} < \frac{6}{20} < \frac{8}{20}$ **and so** $\frac{1}{4} < \frac{3}{10} < \frac{2}{5}$

To practise comparing fractions, a dice game can be used where a fraction is created by throwing two dice to determine a numerator and denominator, then the process is repeated to determine a second fraction. The learner should say which fraction is bigger or explain why the two fractions are equivalent.

Once learners are confident in their understanding of this process, the use of concrete and pictorial approaches can give way to abstract calculations, enabling learners to tackle problems such as the one below.

Stephen has exams in Maths, English and French.
His results are shown below:

English $\frac{12}{20}$ Maths $\frac{27}{40}$ French $\frac{50}{80}$

Put his results in order, starting with his best result.

Possible solution:

English $\frac{12}{20} = \frac{24}{40}$ **Maths** $\frac{27}{40}$ **French** $\frac{50}{80} = \frac{25}{40}$

Therefore Maths is the best result, followed by French then English.

There are a variety of approaches that can be used to compare these results, but finding a common denominator for comparison and conversion to percentages, should both be covered explicitly.

Decimal Fractions

When introducing decimals, it is useful to discuss that a decimal is a fraction written in a different way. Learners could discuss where they have seen decimal fractions used in real life contexts. Examples may include money, sporting times, baking, weight, length etc.

Hundreds	Tens	Ones	Tenths	Hundredths
3	2	7	6	3

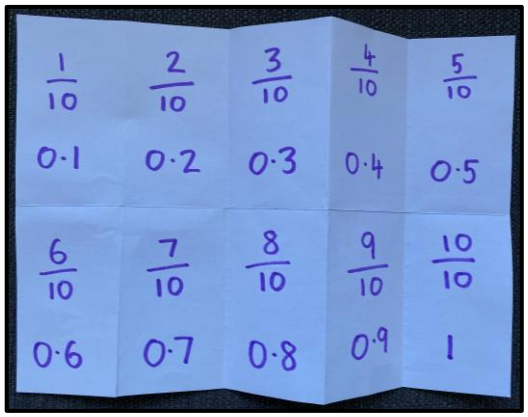
Everything to the left of a decimal point is a whole number.

Everything to the right of a decimal point is part of a whole.

Initially begin by exploring tenths. Learners could fold paper into ten equal parts.

one tenth $\frac{1}{10}$ **0.1**

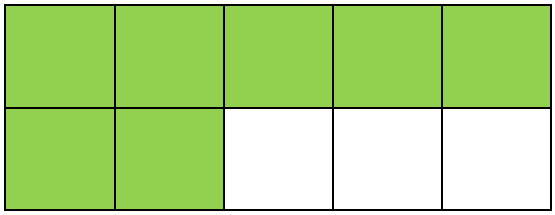
Learners need to understand that these are all different ways of representing the same number.



one tenth	two tenths	three tenths	four tenths	five tenths	six tenths	seven tenths	eight tenths	nine tenths	ten tenths
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

How many tenths are shaded?
What is this as a decimal fraction?

0.7

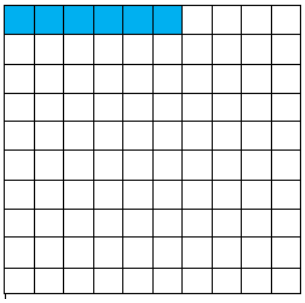


When learners are secure with tenths they can progress to hundredths using a blank hundred square.

one hundredth $\frac{1}{100}$ 0.01

How many squares can you count?

100



Each square is $\frac{1}{100}$.

We can also write this as 0·01

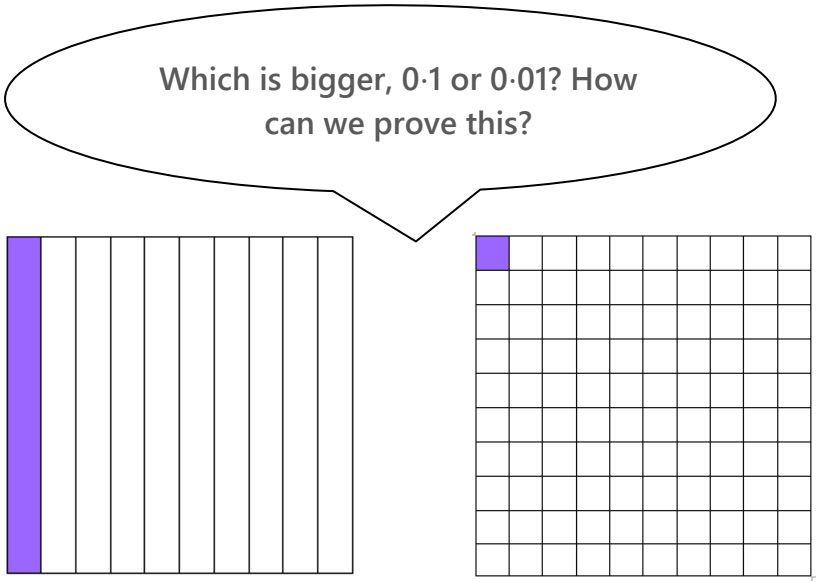
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2
0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.3
0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.4
0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5
0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.6
0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.7
0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.8
0.81	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.9
0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1

In this grid 6 out of one hundred boxes have been coloured.

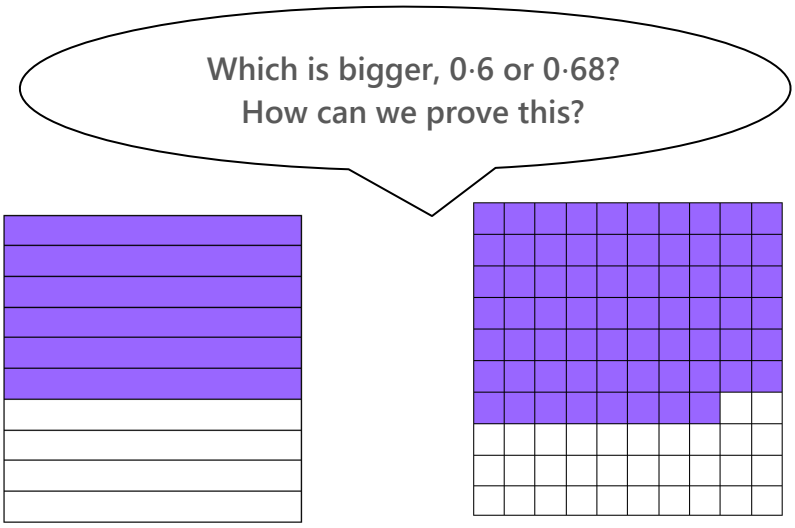
We can write this as $\frac{6}{100}$ or 0·06.

Learners could also be given blank grids and asked to colour a given quantity, e.g. 0·9, 0·78, 0·08 etc. They could also be given partially completed grids and be asked to provide the decimal fraction that has been shaded.

At this stage it is important to discuss the difference between 0.1 and 0.01 using visual examples.



It is also worth exploring that one tenth is the same as ten hundredths using the concrete material.



This can be repeated with a variety of examples and once learners have a sound grasp of this concept using visual examples they can progress to answering similar questions without the visuals.

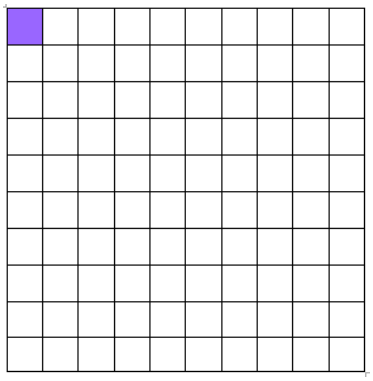
Percentage of a Quantity

Percentages can be introduced when learners have a strong understanding of equivalent fractions. Learners need to understand that decimal fractions and percentages are simply another method for describing a fraction.

> **The sign % stands for 'per cent' which means 'out of 100'.**

It is important to discuss where we might find percentages in everyday life, e.g. shop sales, tax, holiday deposits, etc.

Activities such as using hundred squares can help learners develop a good understanding of the equivalence of simple percentages and fractions.



Out of the 100 squares how may are coloured?

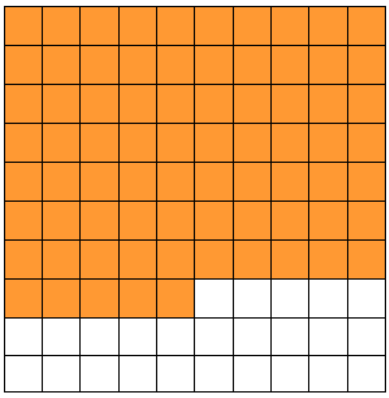
1

We call this 1%

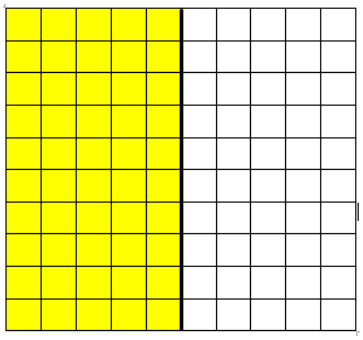
Out of the 100 squares how may are coloured?

75

We call this 75%

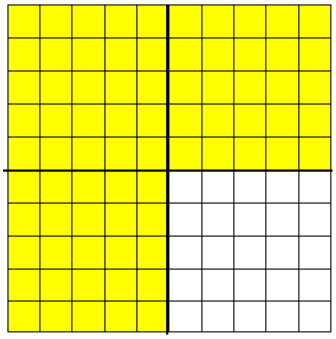


This can be repeated with several similar examples and extended to find equivalent forms of common fraction, decimal fractions and percentages.

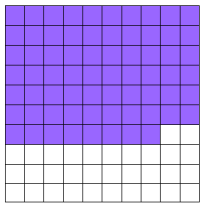


$\frac{50}{100}$ $\frac{1}{2}$ **50%** **0·5**

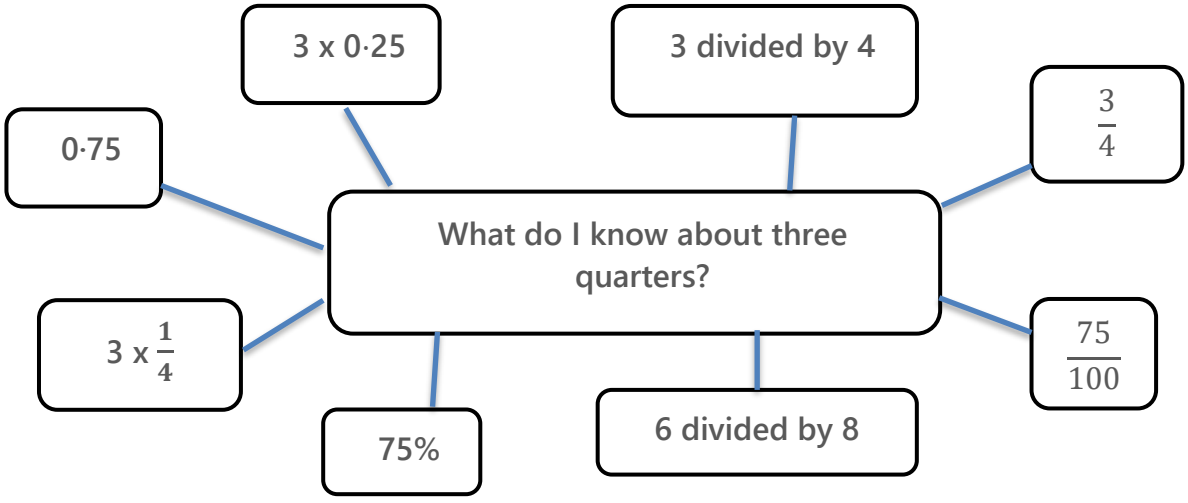
$\frac{75}{100}$ $\frac{3}{4}$ 75% 0·75



Learners can extend their understanding by presenting quantities in different ways.

$\frac{68}{100}$	68%
0·68	

The activity below can provide opportunities for differentiation as learners can access the question at the appropriate level to each individual. Possible answers have been provided but it is likely that learners will provide further possible solutions.



Learners could be asked to put quantities into order starting from smallest to largest.

$\frac{4}{5}$ 22% 0·4 $\frac{1}{4}$ 0·33

Lots of dialogue about the different ways of doing this is very important. Learners can then discuss which way they believe to be the most efficient and why. It is likely that learners will choose to convert all the quantities into decimal fractions or percentages and then put them in order from smallest to largest.

22% → $\frac{1}{4}$ → 0·33 → 0·4 → $\frac{4}{5}$

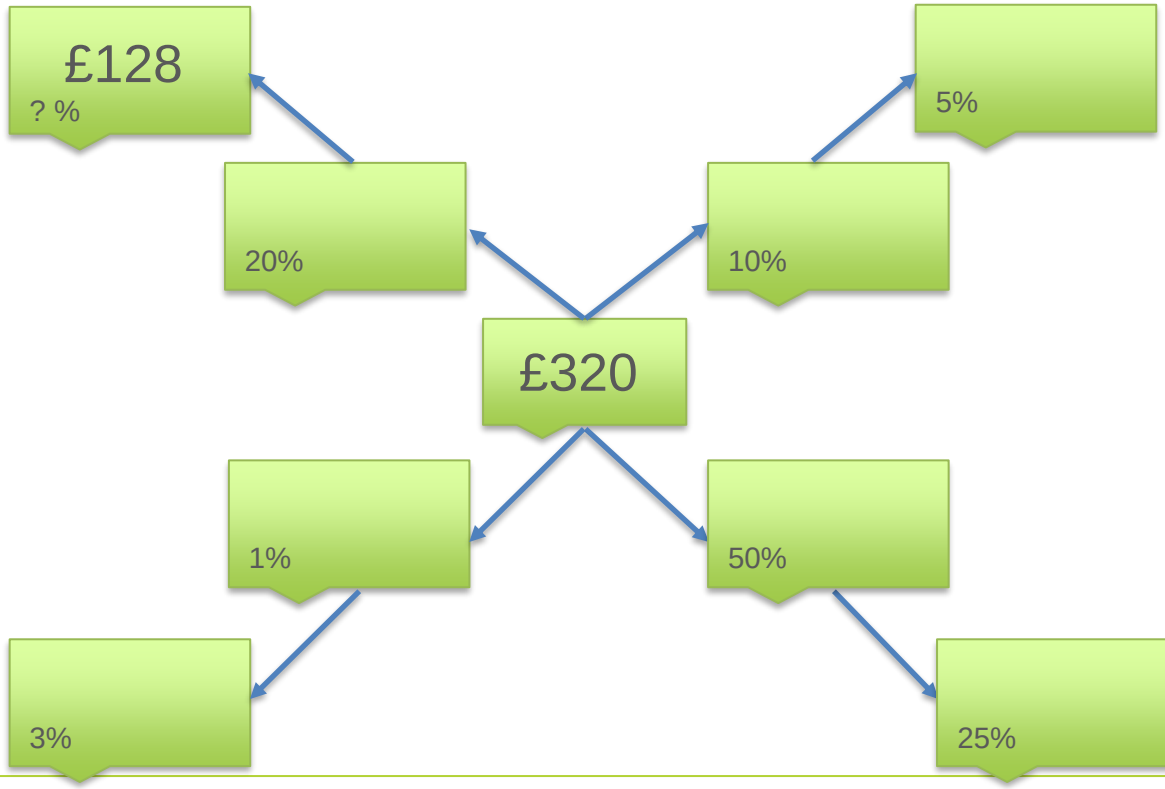
If learners are secure in this equivalence, and in finding a fraction of a quantity, finding simple percentages of quantities is a natural extension of this knowledge. For example to find 25% of 120g, learners can find one quarter of this quantity.

120g			
30g	30g	30g	30g

25% is the same as one quarter. To find one quarter we divide by four. 120g divided by four is 30g.

The above example could also be extended into finding 75% using the knowledge that 75% is the same as three quarters.

Learners should also be given opportunities to use known percentages of quantities to find others. Percentage spider diagrams, such as the one below, can be an effective way to explore this process.



Equivalence of Fractions, Decimal Fractions and Percentages

As learners work through Second level, common equivalences such as $\frac{1}{10}$ = 0•1 = 10% and $\frac{1}{2}$ = 50% = 0•5 become relatively well known, and enable them to answer questions such as:

If one in five people are left-handed, what percentage is this? **20%**

Learners can then move on to examples which involve applying their knowledge of equivalent fractions so they can give their answer in its simplest form.

At a football match, 35% of the crowd were children. **What fraction is this?**
35% = $\frac{35}{100} = \frac{7}{20}$

Learners benefit from a sound understanding of decimal place value when working with decimal fractions. Having the chance to explore equivalences such as 0•4 = $\frac{4}{10} = \frac{40}{100} = \frac{400}{1000}$ and 0•11 = $\frac{11}{100} = \frac{110}{1000}$ can help them to compare fractions with decimal fractions.

Which of the following numbers are less than $\frac{2}{10}$ **?** **0•18 0•208 0•2 0•02** **0·18 and 0·02**

Points to consider

When working with fractions, decimal fractions and percentages, there is a possibility that learners will form misconceptions. Some examples of 'True or False' questions, which can be used to identify misconceptions, are given below. When answering this type of question, it is critical that learners can explain carefully their reasoning and that practitioners plan for how they will address any misconceptions that are uncovered.

True or False: 0•2881 > 0.5	**True or False: 3•7 + 4•5 = 7•12**
True or False: 2•3 x 10 = 2•30	**True or False:** $\frac{1}{10} > \frac{1}{9}$

Links to Other Curriculum Organisers

The use of fractions, decimal fractions and percentages has natural and explicit applications in other parts of the numeracy and mathematics curriculum. This repeated exposure deepens learning and, when highlighted explicitly, can enable learners to see how different aspects of numeracy and mathematics are interconnected. The understanding of fractions, decimal fractions and percentages is linked directly to the following benchmarks.

Number and Number Processes

- Extends knowledge of place value and number processes to decimal fractions. See Number and Number Processes professional learning resource for further information on place value, addition, subtraction, multiplication and division involving decimal fractions.

Multiples, Factors and Primes

- Identifies multiples and factors of whole numbers and applies knowledge and understanding of these when solving relevant problems in number, money and measurement.

Estimation and Rounding

- Rounds decimal fractions to the nearest whole number, to one decimal place and two decimal places. Use also the language of rounding to the nearest tenth and the nearest hundredth.

Money

- Carries out money calculations involving the four operations.

Time

- Knows the relationship between commonly used units of time and carries out conversion calculations, for example changes $1\frac{3}{4}$ hours into minutes.
- Chooses the most appropriate timing device in practical situations and records time using relevant units, including hundredths of a second.

Measurement

- Coverts between common units of measurement using decimal notation, for example, 550 cm = 5•5 m, 3•009 kg = 3009 g.
- Reads a variety of scales accurately.

Ideas of Chance and Uncertainty

- Uses the language of probability accurately to describe the likelihood of simple events occurring, for example, equal chance; fifty-fifty; one in two chance; two in three chance; probability given as a fraction or percentage.

Reflective Questions

- **How can we link new learning to what learners already know?**
- **How can we, as practitioners, develop our own understanding of decimal fractions?**
- **How do we ensure learners build on prior knowledge and skills to extend their understanding of place value when working with whole numbers into working with decimal fractions?**
- **What misconceptions do learners develop and how can we minimise the chances of this happening?**
- **Do we take time to ensure that learners have a robust understanding of the vocabulary associated with fractions, decimal fractions and percentages?**

Third Level

The table below includes the experiences and outcomes and benchmarks related to 'Fractions, Decimal Fractions and Percentages' at third level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	Benchmarks
I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations. **MNU 3-07a** *By applying my knowledge of equivalent fractions and common multiples, I can add and subtract commonly used fractions.* ***MTH 3-07b*** *Having used practical, pictorial and written methods to develop my understanding, I can convert between whole numbers or mixed numbers and fractions.* ***MTH 3-07c*** I can show how quantities that are related can be increased or decreased proportionally and apply this to solve problems in everyday contexts. **MNU 3-08a**	• Converts fractions, decimal fractions or percentages into equivalent fractions, decimal fractions or percentages. • Adds and subtracts whole numbers and fractions, including when changing a denominator • Converts between whole or mixed numbers, improper fractions and decimal fractions. • Uses knowledge of fractions, decimal fractions and percentages to carry out calculations with and without a calculator. • Solves problems in which related quantities are increased or decreased proportionally. • Expresses quantities as a ratio and where appropriate simplifies, for example, 'if there are 6 teachers and 60 children in a school, find the ratio of the number of teachers to the total amount of teachers and children'.

Effective Learning and Teaching Approaches

When learners are ready to move on to third level, the equivalence of simple fractions, decimal fractions and percentages should generally have become known facts. Learners should also have developed strategies to find simple fractions and percentages of quantities. Learners will benefit from regular opportunities to develop their estimation skills and a range of efficient mental calculations for **addition, subtraction, multiplication and division of decimal fractions**.

Equivalence of Fractions, Decimal Fractions and Percentages

Learners benefit from opportunities to develop strategies that use known equivalences to find less common values.

Three out of eight people in a group had blue eyes. What percentage is this?
Possible solution: $\frac{1}{4}$ = 25%, therefore $\frac{1}{8}$ = 12·5%, and so $\frac{3}{8}$ = 37·5%.

Activities that encourage learners to talk about number provide them with the opportunity to identify a wide range of possible approaches and explain their reasoning.

Known or calculated equivalences can be used to compare fractions with different denominators, particularly unfamiliar or unusual fractions. Although non-calculator methods are to be encouraged, it is also important for learners to recognise when using a calculator would be significantly more efficient, and that they can perform an appropriate calculation.

Which is the higher mark, 23 out of 30 or 30 out of 39?
Possible solution: 23 ÷ 30 = 0•76666… 30 ÷ 39 = 0•76923… So 30 out of 39 is the higher mark.

Learners need to know how to convert this information into decimal fractions or percentages, and where necessary, to look to sufficient decimal places to make an appropriate comparison.

Fraction of a Quantity

Learners benefit from developing confidence and fluency in multiplication and division by carrying out calculations involving a wide range of fractions, decimal fractions and percentages, for example:

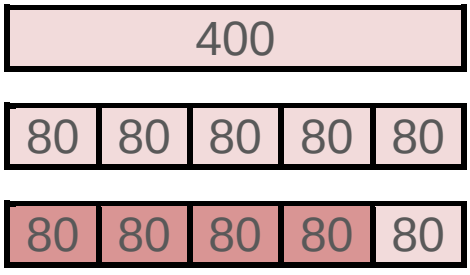
What is $\frac{3}{7}$ of 80•5?	80•5 ÷ 7 = 11.5 11.5 × 3 = 34·5
What is 3•5% of £420?	1% of £420 = £4.20 0•5% of £420 = £2.10 3•5% of £420 = £14.70

It is beneficial to learners if they are also given opportunities to develop their algebraic thinking by solving problems such as:

$\frac{3}{4}$ **of 300 = ?**	$\frac{?}{5}$ **of 600 = 360**	$\frac{2}{3}$ **of ? = 280**	$\frac{3}{?}$ **of 420 = 180**

Bar modelling can be particularly useful in solving problem such as these, for example:

$\frac{?}{5}$ **of 400 = 320**



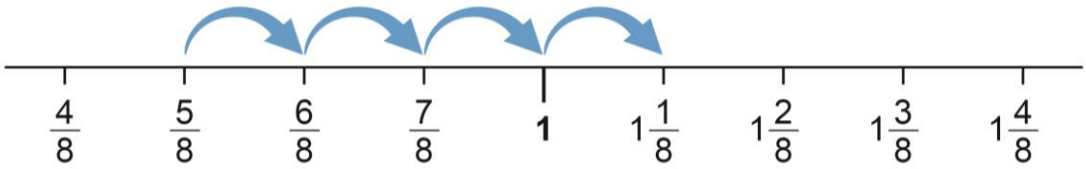
So $\frac{4}{5}$ **of 400 = 320**

Adding and Subtracting Fractions

Number lines can be used to support the development of learners' understanding of adding and subtracting fractions with equal denominators. Learners can also explore fractions that have a value greater than 1 and become familiar with the notation of mixed numbers.

The example below shows that $\frac{5}{8} + \frac{4}{8} = 1\frac{1}{8}$.

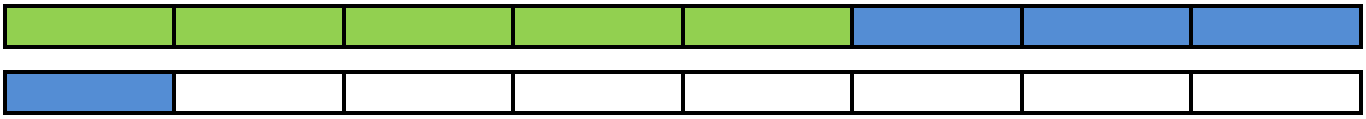
$\frac{9}{8}$ would also be correct and it is beneficial to make time to discuss both answers.



Learners may prefer a pictorial approach:

plus

becomes



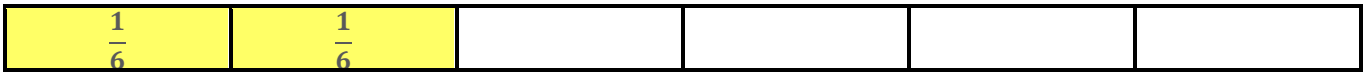
A similar approach can be used illustrate addition and subtraction where the denominators differ. It is important that learners have become confident in finding equivalent fractions before they move on to this stage.

The example below shows that $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

plus

is equivalent to

plus



and becomes

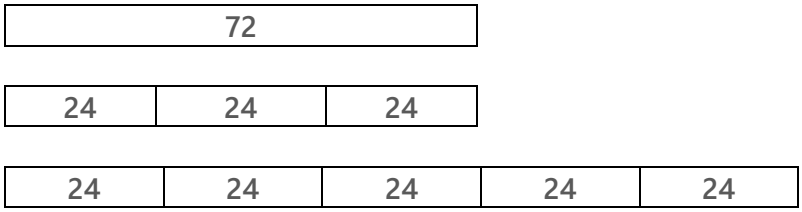
To practise adding fractions, a dice game can be used where a fraction is created by throwing two dice to determine a numerator and denominator, then the process is repeated to determine a second fraction to be added. Dice throws may result in an improper fraction, but limiting the numbers to six and below keeps the numerical complexity down and allows learners to focus on the new learning. To extend this activity to include subtraction, learners would either have to identify the larger fractions to determine the order of the subtraction, or they could explore negative fractions as an extension activity.

Ratio and Proportion

A variety of problems across the numeracy and mathematics curriculum can be solved using proportional reasoning, and so it is beneficial for learners to use reliable strategies to perform such calculations. Learners can apply their multiplication and division strategies to solve problems such as:

3 tins of paint are used to cover an area of 72 square metres.

What area will 5 tins cover?

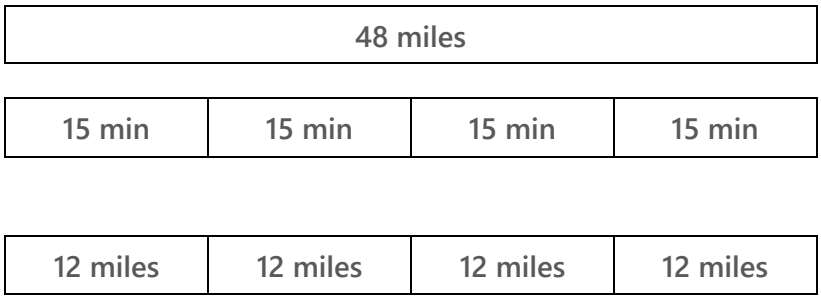


5 tins will cover an area of 120m^2.

Learners may choose to use proportional reasoning to solve problems relating to speed, distance and time for example

A car was travelling at an average speed of 48 miles per hour.

How far did it travel in 15 minutes?



It travels 12 miles in 15 minutes.

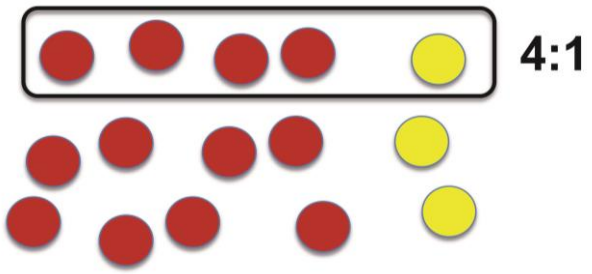
Learners also use these skills when tackling problems involving currency conversions, and finding the best value when comparing prices.

Concrete and pictorial approaches can be used to illustrate ratios and their simplification.

There are 12 red counters and 3 yellow counters.

Work out the ratio of red counters to yellow counters.

Write the ratio in its simplest form.



Links to Other Curriculum Organisers

The use of fractions has natural and explicit applications within other parts of the Numeracy and Mathematics curriculum. This repeated exposure deepens learning and, when highlighted explicitly, can enable learners to see how different aspects of numeracy and mathematics are interconnected. The understanding of fractions, decimal fractions and percentages is linked directly to the following benchmarks.

Number and Number Processes

- Extends knowledge of place value and number processes to decimal fractions. See Number and Number Processes professional learning resource for further information on place value, addition, subtraction, multiplication and division involving decimal fractions

Multiples, Factors and Primes

- Identifies common multiples, including the lowest common multiple for whole numbers and can explain method used.
- Identifies common factors, including the highest common factor for whole numbers and can explain method used.

Money

- Demonstrates understanding of best value in relation to contracts and services when comparing products.
- Chooses the best value for their personal situation and justifies choices.
- Converts between different currencies.

Time

- Applies knowledge of the relationship between speed, distance and time to find each of the three variables.

Reflective Questions

- **How do we enable learners to find efficient strategies for finding fractions and percentages of quantities?**

- **How can we use pictorial approaches to support learners to develop a deep understanding of addition and subtraction of fractions?**

- **How do we make explicit the similarities and differences between fractions and ratios?**

Fourth Level

The table below includes the experiences and outcomes and benchmarks related to 'Fractions, Decimal Fractions and Percentages' at fourth level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Curriculum Organisers	Benchmarks
I can choose the most appropriate form of fractions, decimal fractions and percentages to use when making calculations mentally, in written form or using technology, then use my solutions to make comparisons, decisions and choices. **MNU 4-07a** *I can solve problems involving fractions and mixed numbers in context, using addition, subtraction or multiplication.* ***MTH 4-07b*** Using proportion, I can calculate the change in one quantity caused by a change in a related quantity and solve real-life problems. **MNU 4-08a**	• Chooses the most efficient form of fractions, decimal fractions or percentages when making calculations • Uses calculations to support comparisons, decisions and choices • Applies addition, subtraction and multiplication skills to solve problems involving fractions and mixed numbers • Uses knowledge of proportion to solve problems in real-life which involve changes in related quantities.

Effective Learning and Teaching Approaches

When learners are ready to move on to fourth level, they should have developed a variety of strategies for working with fractions, decimal fractions and percentages.

Multiplication and Division of Fractions

It is important that conceptual understanding underpins the teaching of procedural calculations. The use of known facts and relationships to deduce new insights can also be a powerful tool when introducing new concepts. In the example on the next page, known multiplication and division facts are used to explore related calculations involving fractions.

$12 \times 4 = 48$

$12 \times 2 = 24$

$12 \times 1 = 12$

$12 \times \frac{1}{2} = ?$

$12 \times \frac{1}{4} = ?$

$6 \times \frac{1}{4} = ?$

$3 \times \frac{1}{4} = ?$

$\frac{3}{2} \times \frac{1}{4} = ?$

Can you see any patterns in the questions?

Try to describe any patterns you see in the answers.

Can you explain what it means to multiply by one quarter?

Can you explain what is happening when we divide by a half?

Can you draw what is happening when we divide by a half?

$12 \div 4 = 3$

$12 \div 2 = 6$

$12 \div 1 = 12$

$12 \div \frac{1}{2} = ?$

$12 \div \frac{1}{4} = ?$

$6 \div \frac{1}{4} = ?$

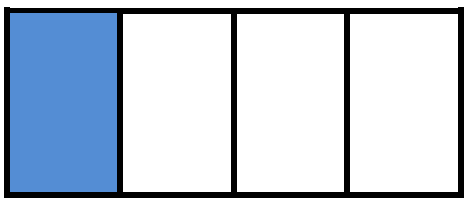
$3 \div \frac{1}{4} = ?$

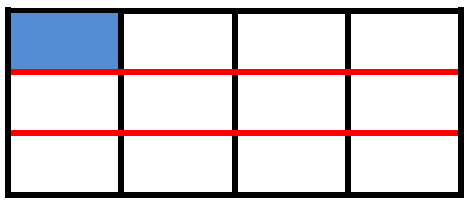
$\frac{3}{2} \div \frac{1}{4} = ?$

Pictorial approaches, in particular arrays, can be used to illustrate multiplication and division of fractions. With multiplication, the overall concept of “a fraction of a fraction” is important and the concept of commutativity can be reinforced.

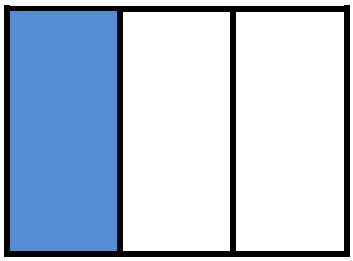
For example, to show $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

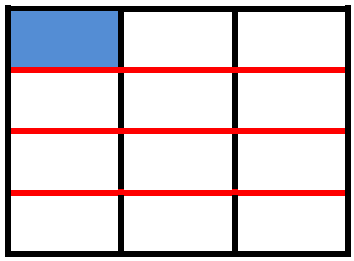
Split one quarter into thirds:



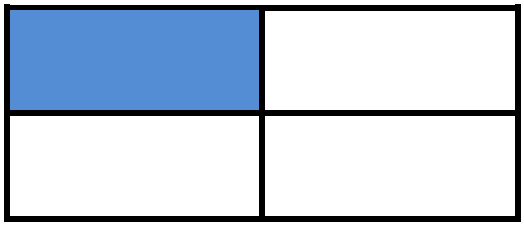


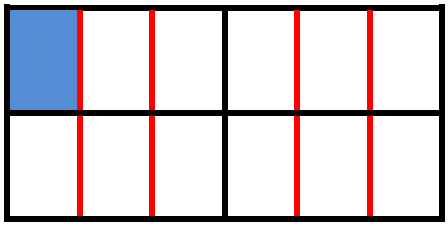
Alternatively, split one third into quarters.



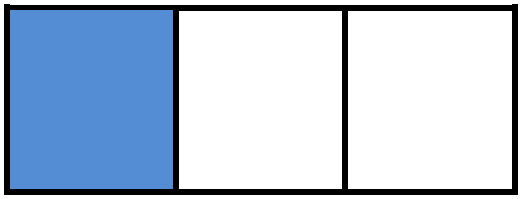


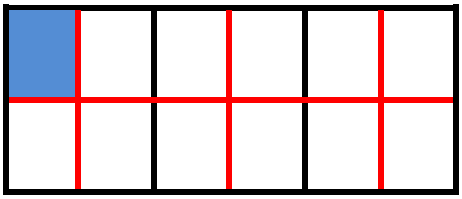
There are alternative ways to represent this calculation, for example splitting one quarter into thirds





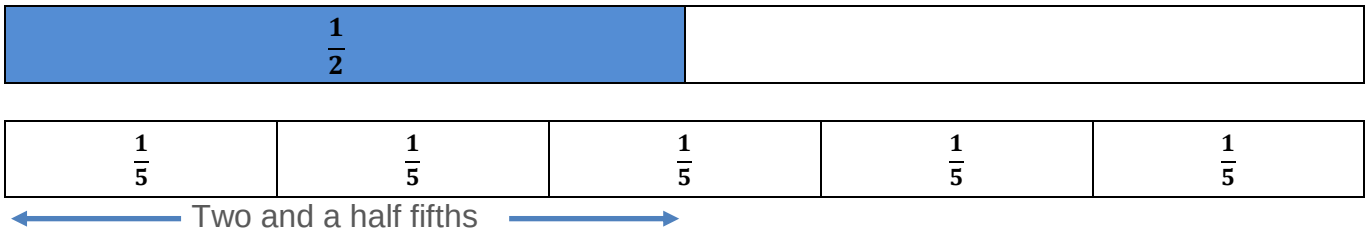
and splitting one third into quarters.



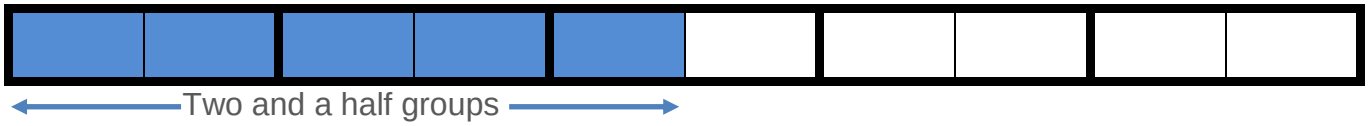


With division, the overall concept of “how many fractions are in a fraction?” is important.

For example, to show $\frac{1}{2} \div \frac{1}{5} = 2\frac{1}{2}$, consider “how many fifths are in one half?”



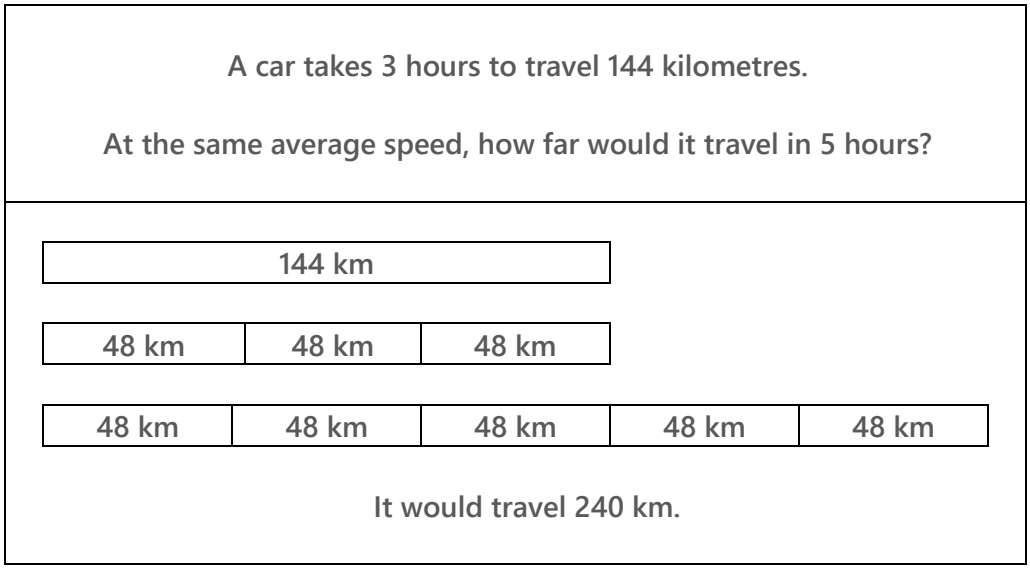
This pictorial approach can be adjusted slightly to build on the prior knowledge of equivalent fractions, reframing the question as “how many groups of two tenths are in 5 tenths?”



These pictorial approaches can be replaced with abstract processes when learners become confident in their understanding.

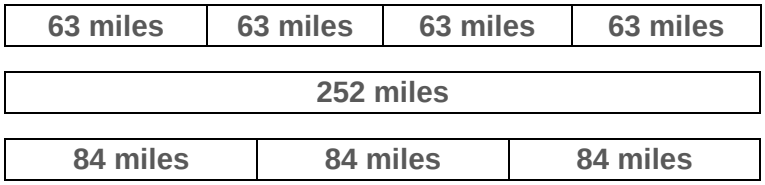
Proportion

Bar models can be helpful to learners when they are introduced to the concepts of direct **and** indirect proportion. Distance, speed and time can provide a familiar context for such calculations.



A train takes 4 hours to complete a journey at an average speed of 63 miles per hour.

If another train completes the same journey in 3 hours, what is its average speed?



Its average speed is 84 miles per hour.

Links to Other Curriculum Organisers

The use of fractions, decimal fractions and percentages has natural and explicit applications in other parts of the numeracy and mathematics curriculum. This repeated exposure can deepen learning and, when highlighted explicitly, can enable learners to see how different aspects of numeracy and mathematics are connected.

At fourth level, learners benefit from solving a variety of multi-step problems which require an understanding of fractions, decimal fractions and percentages which involve making comparisons and decisions, and communicating their reasoning. Money, measurement, shape and time provide natural contexts for such problems and reinforce the need to round answers to an appropriate degree of accuracy. Trigonometry and similar shapes provide explicit contexts for ratio.

Reflective Questions

How do we ensure our learners deepen their understanding of fractions whilst becoming proficient in performing calculations?

How do we ensure learners are applying their skills across a variety of topics in solving multi-step problems?

How are we ensuring our learners develop confidence and accuracy in mental and written calculations?

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