Ideas of Chance and Uncertainty

Professional Learning Resource

This resource is part of the suite of Numeracy Professional Learning Resources.

For Scotland's learners, with Scotland's educators

Do luchd-ionnsachaidh na h-Alba, le luchd-foghlaim Alba

Contents

Introduction

This professional learning resource has been created to enable practitioners to reflect on their own knowledge and understanding and outline effective approaches to support future learning and teaching in Ideas of Chance and Uncertainty.

Using everyday language to identify outcomes of familiar events supports the development of critical thinking skills. This enables discussion around choices and consideration of alternative options when making decisions.

Predicting and describing the likelihood of events occurring can help develop mathematical thinking skills and the ability to make informed choices.

National Numeracy and Mathematics Progression Framework[1]
Ideas of Chance and Uncertainty

[1] Further guidance on using the National Numeracy and Mathematics Progression Framework can be found by clicking here.

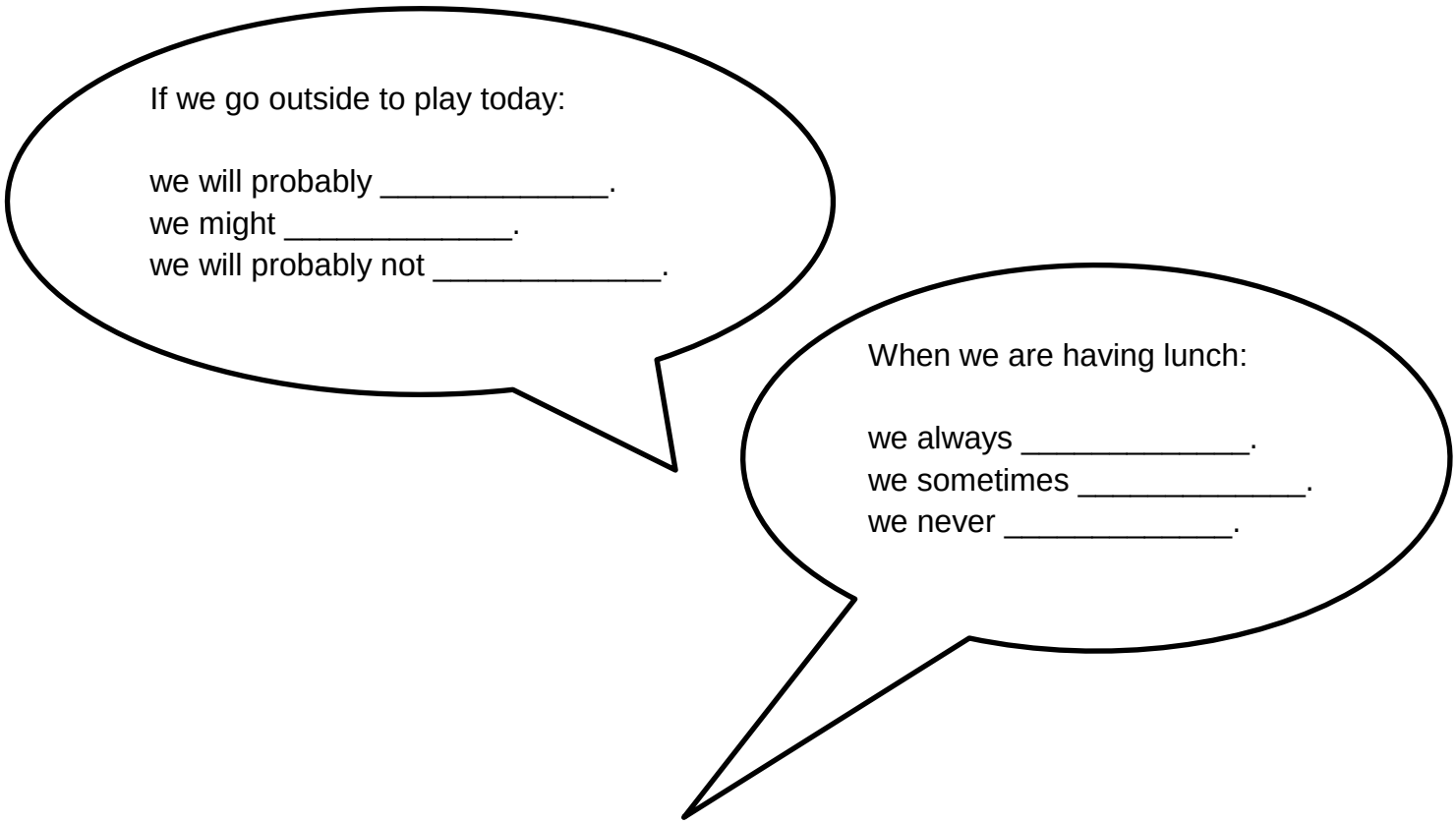
Early Level

Experiences and Outcomes	Benchmarks
Ideas of Chance and Uncertainty	
There are no experiences and outcomes at early level.	

Effective Learning and Teaching Approaches

Whilst there are no formal experiences and outcomes appearing at early level, learners can begin to gain an understanding of the idea of chance and uncertainty through the use of appropriate vocabulary. Practitioners are encouraged to model the use of vocabulary such as likely and unlikely, never, always, fair and unfair in daily routines, events and learning opportunities. Building on awareness of what these terms mean through informal situations will allow pupils to begin to understand these concepts.

Posing different questions and thinking about how likely or unlikely possible answers will be is one way to develop vocabulary.



First Level

The table below includes the experiences and outcomes related to 'Ideas of Chance and Uncertainty' at first level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	**Benchmarks**
Ideas of Chance and Uncertainty	
I can use appropriate vocabulary to describe the likelihood of events occurring, using the knowledge and experiences of myself and others to guide me. **MNU 1-22a**	• Uses mathematical vocabulary appropriately to describe the likelihood of events occurring in everyday situations including, probable, likely/unlikely, certain/uncertain, possible/impossible, and fair/unfair. • Interprets data gathered through everyday experiences to make reasonable predictions of the likelihood of an event occurring

Effective Learning and Teaching Approaches

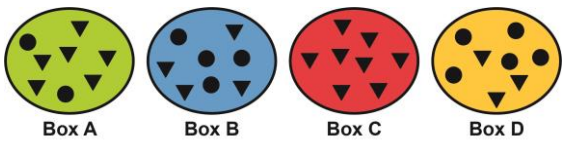
It is important that learners meet an introductory level of the vocabulary of chance such as, likely and unlikely, never, always, certain and impossible. Learners should be encouraged to share the language they already know and start to develop an understanding of order of likelihood.

Confident learners have a good understanding of an introductory level of vocabulary of chance.

Maria takes a shape out of a box without looking.

It is **certain** that the shape is a triangle.

Which box is it?

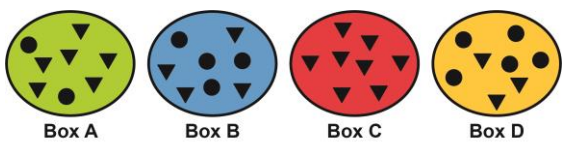


Understanding of concepts such as probable, possible, equally likely and the application of more than one condition should be further developed.

Maria takes a shape out of a box without looking.

It is **very likely** but not certain that the shape is a triangle.

Which box is it?



Tasks should be designed to encourage learners to discuss issues associated with probability.

Examples

You ask a family member if you can go to the swimming pool on Saturday morning. What sort of things might they say in reply? Can you pick out the words that tell you how likely it is that you will actually go to the pool? Can you arrange your words in order from ‘never’ at one end to ‘definitely’ at the other?

Games using dice or spinners can be used to introduce the idea of fairness. Many learners (and adults) think that a six is harder to throw than any other number on a die, this could be because in their experience they often need to throw a six to start or win a game. This perception could be challenged by playing games using dice and spinners and talk about if each game was fair and how they know.

Learners could be asked in groups to discuss whether they agree or disagree with a range of statements.

- The hardest number to throw on a die is six.
- A game is fair if you follow the rules.
- It always rains during the school holidays.
- It is easier to get ‘tails’ when you flip a coin.
- If you buy lots of raffle tickets, you are sure to win a prize.

It is important that learners are encouraged to explain their thinking.

Using examples of data is another way that learners can be encouraged to discuss the likelihood of events occurring and make predictions. Some examples of data that could be used are listed below. It is important that data is relevant to the learners and is in a context with which they are familiar.

weather forecast	rainfall in the garden	snack time choices
voting outcomes	football league table	school award winners
sports day results	family pets	shopping receipts

Things to consider

- Learners experiences of playing games involving dice and spinners will be varied.
- Learners may find it more challenging to solve problems where more than one condition is given or where terms such as possible or probable are used.
- It is important that examples are set in contexts which are familiar to learners.

Links to Other Curriculum Areas

Time

- Knows the months of the year and talks about features of the four seasons in relevant contexts.

Mathematics and its impact on the world, past, present and future

- Investigates and shares understanding of how mathematical predications are used in everyday situations. For example, choosing an activity based on weather forecasting or forming strategies in games.

Reflective Questions

- **At this level it is important to make connections to using the vocabulary of chance and uncertainty in daily life. How is this being addressed?**
- **How can we encourage all adults in school to model the mathematical vocabulary to learners through everyday situations and events?**
- **How are we making use of practical activities, such as games using dice or spinners to develop learners' understanding?**
- **What everyday data can be used to discuss likelihood and make predictions?**

Second Level

The table below includes the experiences and outcomes related to 'Ideas of Chance and Uncertainty' at second level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	**Benchmarks**
Ideas of Chance and Uncertainty	
I can conduct simple experiments involving chance and communicate my predictions and findings using the vocabulary of probability. **MNU 2-22a**	• Uses the language of probability accurately to describe the likelihood of simple events occurring, for example equal chance; fifty-fifty; one in two, two in three; percentage chance; and $\frac{1}{6}$. • Plans and carries out simple experiments involving chance with repeated trials, for example, 'what is the probability of throwing a six if you throw a die fifty times?'. • Uses data to predict the outcome of a simple experiment.

Effective Learning and Teaching Approaches

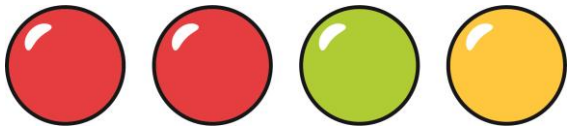
It is important that learners are given opportunities to investigate probability by conducting practical activities. The development of appropriate mathematical language will enable learners to make sense of the results of these activities.

Learners need to develop an understanding that, in terms of probability, an experiment is a repeated procedure with a set of possible outcomes. Learners should develop the ability to identify all the possible outcomes, record and organise the results of their experiment and to discuss and describe their findings.

At this level learners should be beginning to extend their understanding of the language of probability and to assign numerical values to the likelihood of a simple event occurring.

If a marble is chosen at random, what is the probability the chosen counter will be green?

One in four chance



It is important that learners understand the difference between theoretical probability and experimental probability. To help learners understand this difference it could be explained that theoretical probability is what should happen in an ideal world and experimental probability is what actually happens in the experiment.

For example, if a coin is flipped 50 times, there are two equally likely outcomes. Theoretically we would expect 25 heads and 25 tails, however when the experiment is carried out the result may be 23 heads and 27 tails. By increasing the number of times that the experiment is repeated, learners will be able to see that the experimental probability generally gets closer to the theoretical probability.

Examples

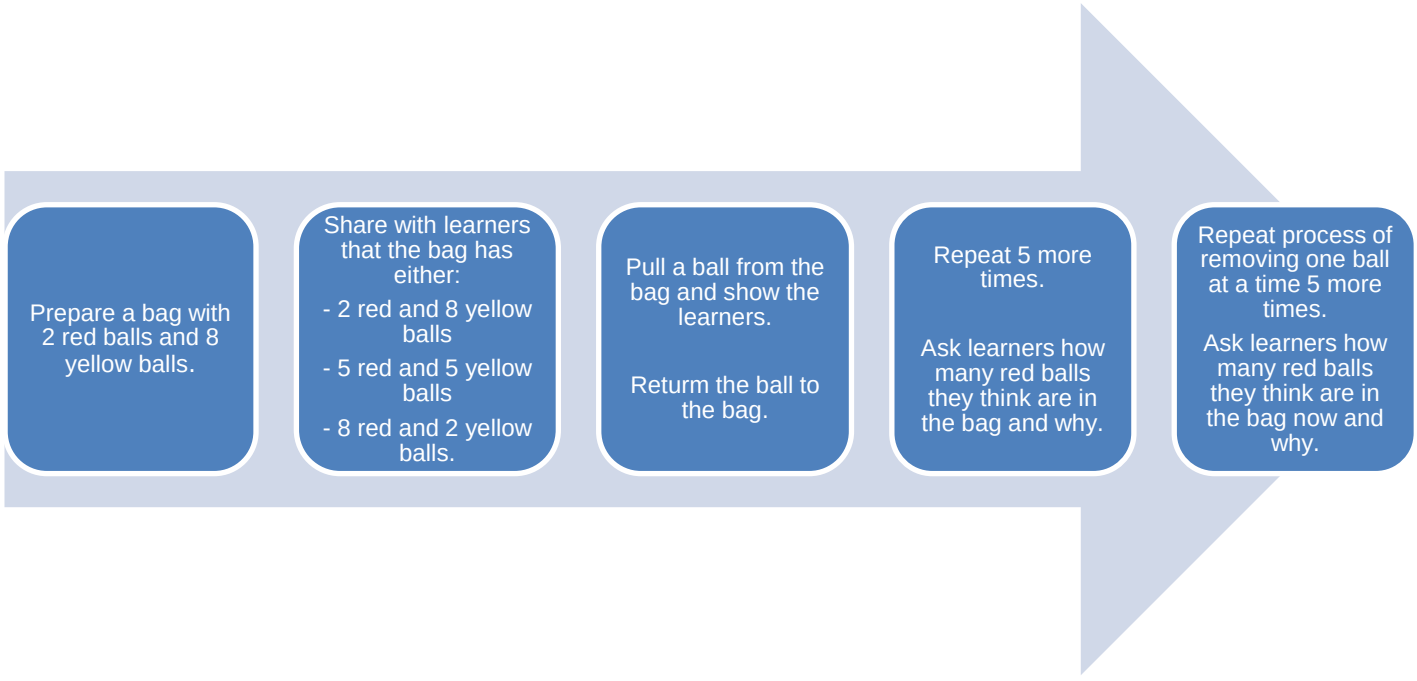
Simple experiments could involve some of the examples below.

- Rolling a die, a set number of times and recording the results.
- Flipping a coin a set number of times and recording how many times it lands each of heads and tails.
- Using and recording results from a variety of spinners.
- Making use of digital spinners and dice.

These could be set up as stations across the classroom to ensure that learners experience a variety of different ways of recording results.

Sets of numbered cards can be used to make predictions related to number properties such as the chance of picking an odd (or even) number, a number greater than 5, a multiple of 3 or a factor of 20.

It is also important to provide the opportunity to extend their thinking, for example:



Once the correct bag has been established the learners could be encouraged to make links to their fractions knowledge, for example understanding that the chance of pulling out a red ball is 2 out of 10 or one fifth.

Things to consider

- Learners require a good understanding of fractions before they can assign numerical values to the probability of an event.
- Continued use of practical activities will be needed to ensure that learners can calculate probability in a variety of contexts.
- Some learners may need support to record and organise the results of their experiments.

Links to Other Curriculum Areas

Multiples, factors and primes

- Applies knowledge and understanding of multiples and factors when solving probability questions.

Fractions, decimal fractions and percentages

- Assigns a numerical value to a probability and expressing it in fraction form.

Data and analysis

- Collect, organise and analyse experimental data, making use of technology if appropriate.

Reflective Questions

- **How can we develop understanding of probability for learners?**
- **How can we facilitate experimental probability in our setting?**
- **How can we make use of digital technology to develop learners' chance and uncertainty skills?**

Third Level

The table below includes the experiences and outcomes related to 'Ideas of Chance and Uncertainty' at third level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	**Benchmarks**
Ideas of Chance and Uncertainty	
I can find the probability of a simple event happening and explain why the consequences of the event, as well as its probability, should be considered when making choices. **MNU 3-22a**	• Uses the probability scale of 0 to 1 showing probability as a fraction or decimal fraction. • Demonstrates understanding of the relationship between the frequency of an event happening and the probability of it happening. • Uses a given probability to calculate an expected outcome, for example, 'the probability of rain in June is 0·25 so how many days do we expect it to rain?'. • Calculates the probability of a simple event happening, for example, 'what is the probability of throwing a prime number on a 12 sided die?'. • Identifies all of the mutually exclusive outcomes of a single event and calculates the probability of each. • Investigates real-life situations which involve making decisions on the likelihood of events occurring and the consequences involved.

Effective Learning and Teaching Approaches

Learners at this level should be given the opportunity to solve a variety of problems in familiar and less familiar contexts which require them to calculate the probability of an event using the definition.

$$Probability\ of\ event = \frac{Number\ of\ favourable\ events}{Total\ number\ of\ possible\ events}$$

The calculation of probability gives a rich opportunity to revisit and consolidate work with fractions, decimal fractions, percentages and their equivalences. It is important that learners can relate probabilities expressed in each form to each other, to the language of probability and to real life events.

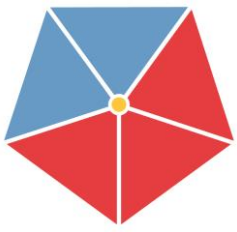
Impossible	Even Chance	Certain
0	0.5	1
0	$\frac{1}{2}$	1
0%	50%	100%
Picking a red counter out of a bag that only contains yellow counters	Rolling an even number on a dice	July is the month after June

Learners should be given the opportunity to choose the most appropriate form, taking into consideration accuracy and clarity of communication. Examples of how probability is expressed in real life applications could be discussed, for example, in weather forecasting the probability of rain is usually expressed as a percentage and the chance of winning a prize from a game is often displayed as a ratio.

It is important that learners have a clear understanding of all mutually exclusive outcomes of a single event and are able to identify these. For example, if one standard die is thrown then there are six mutually exclusive outcomes. Each outcome is mutually exclusive as it is not possible to roll both a one and a two at the same time.

It is important that learners understand that the total of the probabilities of each mutually exclusive event is 1 as it is certain that one of these possible outcomes will occur when the die is rolled. Learners could be asked to investigate by calculating the probability of each mutually exclusive outcome, finding the total and then discussing why this is the case.

When learners can confidently list all mutually exclusive outcomes of a simple event, they should be given the opportunity to investigate different events. For example, learners could investigate a pentagonal spinner which has three red sections and two blue sections to find the probability of **not** landing on a particular colour.

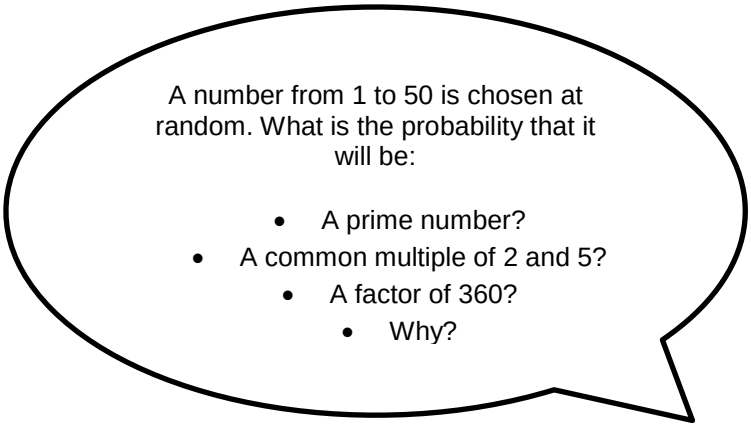


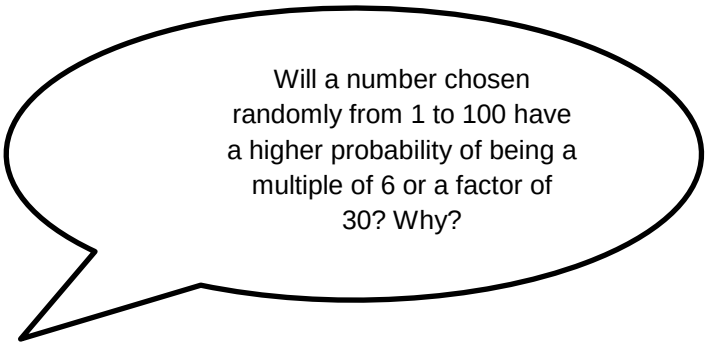
Mutually exclusive outcomes:
Red, red, red, blue, blue

$P(Blue) = \frac{2}{5}$ $P(Not\ blue) = \frac{3}{5}$

$P(Yellow) = 0$

Carefully constructed probability questions could provide the opportunity to consolidate work on multiples factors and primes.





It is important that learners continue to have the opportunity to carry out practical activities to support their learning. The relationship between theoretical and experimental probability can be further explored to enhance understanding of the connection between frequency of an event happening and its probability.

Learners could work in groups or pairs to trial the same experiment and then results can be combined to demonstrate that as the number of trials increases the experimental probability becomes closer to theoretical probability. Digital technology such as spreadsheets, could be used to collate and display results.

An understanding of the link can then be used to calculate the expected frequency of an outcome when the theoretical probability is known or given.

It is important to link work on chance and uncertainty to real life situations and familiar contexts. Discussion could be encouraged by showing learners freely available media content relating to real-life situations which involve making decisions on the likelihood of events occurring and the consequences involved. Examples could include:

- weather forecasting
- earthquake prediction
- insurance premiums
- micromorts – a measure of the risk of day to day activities.

Things to consider

- Learners will require a good understanding of equivalent fractions, decimal fractions and percentages when solving probability problems.
- Continued use of practical activities will be needed to ensure that learners can calculate probability in a variety of contexts.
- Learners need time to discuss and may require support to identify all possible mutually exclusive outcomes before being asked to calculate the probability of an event.
- A large number of repetitions of an experiment may be required to demonstrate the link between theoretical and experimental probability.
- Digital technology could be used to record and display results and support understanding.

Links to Other Curriculum Areas

Estimation and rounding

- Round a number using an appropriate degree of accuracy, having taken into account the context of the problem.

Multiples, factors and primes

- Solve well-constructed probability questions by applying knowledge and understanding of common multiples, common factors and primes.

Fractions, decimal fractions and percentages

- Probability questions provide an opportunity to work with equivalent fractions, decimal fractions and percentages, carry out calculations and make comparisons and informed choices in real life situations.

Data and analysis

- Organise and interpret data, making use of technology if appropriate.

Reflective Questions

- **How often and in what ways are learners involved in practical activities which support understanding of chance and uncertainty?**
- **What real life situations and decision making can we link to probability tasks?**

Fourth Level

The table below includes the experiences and outcomes related to 'Ideas of Chance and Uncertainty' at fourth level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	**Benchmarks**
Ideas of Chance and Uncertainty	
By applying my understanding of probability, I can determine how many times I expect an event to occur, and use this information to make predictions, risk assessment, informed choices and decisions. **MNU 4-22a**	• Calculates the probability and determines the expected occurrence of an event. • Applies knowledge and skills in calculating probability to make predictions.

Effective Learning and Teaching Approaches

Learners should be given the opportunity to use their knowledge of probability to solve increasingly complex problems, sometimes in less familiar contexts. Problems could involve combined and linked events.

Learners could be asked to predict the probability of getting two heads when flipping two coins, they could then be asked to carry out an experiment to test the supposition and finally they could be asked to identify all possible mutually exclusive outcomes and calculate the theoretical probability of getting two heads.

Coin 1 / Coin 2	Head(H)	Tail(T)
Head(H)	H,H	H,T
Tail(T)	T,H	T,T

Learners could be asked to record all possible mutually exclusive outcomes when rolling two dice simultaneously. It important to encourage a systematic approach to the listing of all outcomes. The use of a table could be encouraged.

		Die 1					
		1	2	3	4	5	6
Die 2	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

When learners have identified all possible mutually exclusive outcomes a number of questions could be posed. Some examples include:

- What is the probability of rolling a total of less than 5?
- Which total has the highest probability?
- Which total is least likely?
- What is the probability of throwing a double?

Learners should be encouraged to justify their answers.

A number of activities using a set of shuffled cards numbered from 1 to 10 could be used to investigate the probability of linked events.

- If the first card turned over is a 7, what is the probability that the next card will have a higher value?
- Cards with values 1, 3 and 6 are turned over. What is the probability that the next card will be an even number less than 8?

Further activities and questions could be created using a standard set of playing cards. Practitioners should be aware that not all learners will be familiar with playing cards and these activities should, in the first instance, use physical playing cards before using digital versions.

Problems should be constructed which allow learners to determine the expected occurrences of an event using a given or calculated probability. It is important that learners are encouraged to communicate their predictions clearly and in the context of the question, for example learners could be asked to consider the records of a local vet which show which types of pets they have seen in their surgery over the course of a typical month.

Pet	Dog	Cat	Rabbit	Hamster	Snake	Other
Frequency	58	47	18	12	3	2

Over the course of the year they see 7800 pets. Use the information provided to predict how many will be rabbits.

It is important that learners are given the opportunity to investigate and discuss how probability can be used in real life to assist us to make informed choices and decisions. Practitioners could make use of real-life statistics available online to stimulate discussion, such as health, transport or sport. Learners should be given the opportunity to understand that, as with graphs and charts, probability statistics can be presented in a misleading way.

For example, a headline news article stating that a new drug can reduce the risk of heart attacks by 40% could be investigated to enable learners to fully understand what this means.

If the risk of having a heart attack in the next 10 years was presently 10% then a 40% reduction would bring this down to 6%. To help visualise this we could think about this in the context of number of people, if there were 100 people who each had a 10% risk then the 40% reduction would mean that 6 out of the hundred would have a heart attack instead of 10.

This means that the drug prevents a heart attack for $\frac{4}{100}$ or $\frac{1}{25}$ of the population while the other $\frac{24}{25}$ would gain no benefit. Learners could then be encouraged to discuss this and consider what other factor might need to be taken into account when making decisions about prescribing the new drug.

Things to consider

- Activities using a pack of standard playing cards will not be a familiar context for all learners**.**

Links to Other Curriculum Areas

Fractions, decimal fractions and percentages

- Choose the most efficient form of fractions, decimal fractions or percentages when making calculations and uses calculations to support comparisons, decisions and choices.

Reflective Questions

- **How often, and in what ways, are learners involved in practical activities which support understanding of chance and uncertainty?**
- **Are we linking probability to increasingly complex real-life situations and decision making? Could this be further developed?**
- **In what ways are we making use of digital media to stimulate rich discussion?**

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