Measurement

Professional Learning Resource

This resource is part of the suite of the Numeracy Professional Learning Resources

February 2024

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Introduction

This professional learning resource has been created to enable practitioners to reflect on their own knowledge and understanding, highlight areas which learners find challenging and outline effective approaches to support future teaching and learning in measurement.

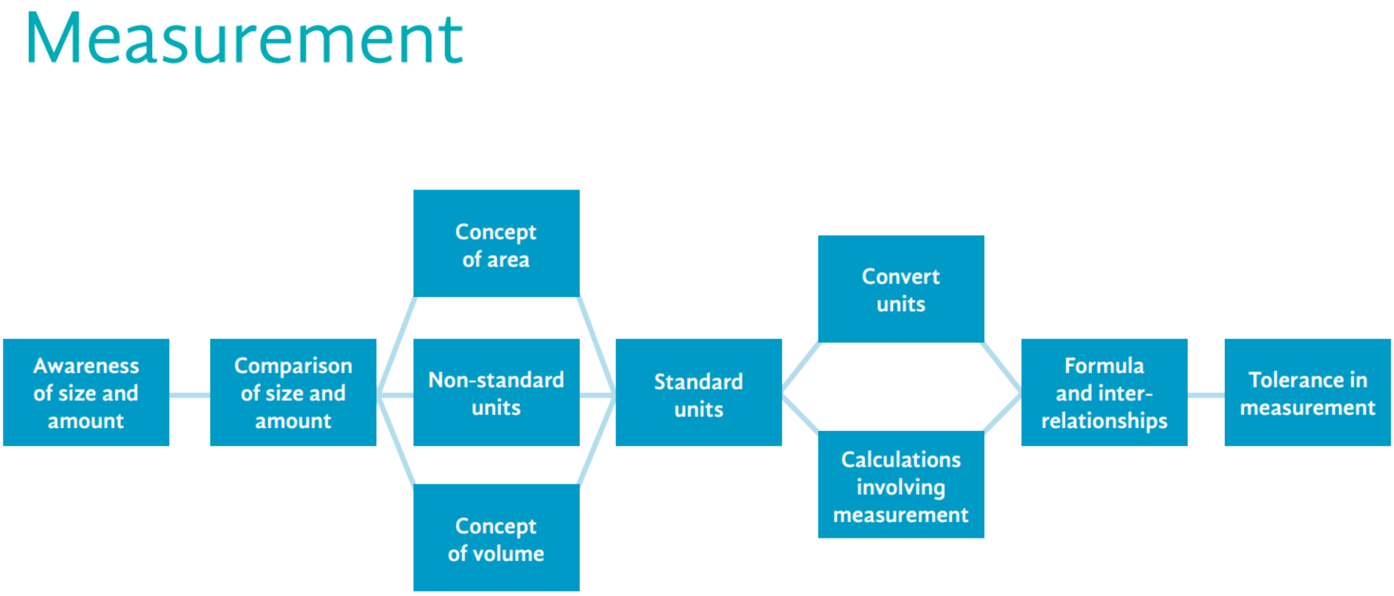
Measuring is assigning a number to an object in terms of how much of the attribute it has. Examples include, but are not limited to, length, height, mass and capacity. Measurement activities will provide opportunities for learners to consolidate and extend their number knowledge and skills.

Teaching and learning about measurement can provide rich and meaningful contexts for the use of estimation, number skills and spatial concepts. The skills we are supporting learners to develop will go on to help them in everyday life, for example, when cooking, decorating, keeping healthy and travelling.

Throughout all levels learners should be provided with opportunities to apply their understanding of measurement in real life situations.

In real life, estimation is part of our everyday experiences and it is an essential skill for learners to develop as it will support them to determine the reasonableness of any measurements that they make. Before carrying out any type of measurement activity, learners should first be asked to make an estimate of what the item they are measuring might be. Opportunities should also be provided for learners to consider the most appropriate measuring device and unit of measurement.

National Numeracy and Mathematics Progression Framework[1]
Measurement



[1] Further guidance on using the National Numeracy and Mathematics Progression Framework can be found by clicking here.

There are a wide range of terms in relation to measurement.

Length/ distance	The distance from one point to another. E.g., How long something is from end to end. Often measured in millimetres (mm), centimetres (cm), metres (m) and kilometres (km).
Height	How tall something is from its base to its top. Measured similarly to length.
Mass (sometimes referred to as weight)	Mass is a measure of the amount of matter in an object and is often measured in grams (g) and kilograms (kg). Weight is a measure of the size of the pull of gravity on the object and is measured in newtons (N). Mass and weight are often used interchangeably because gravity is fairly constant on Earth. If you were to travel to the moon, your mass would remain the same but your weight would change due to the change in gravitational pull.
Capacity	The amount an object or container can hold. Often measured in millilitres (ml) or litres (l).
Volume	The measure of space taken up by a three dimensional object. Volume is measured in cubic centimetres (cm^3) and cubic metres (m^3). It is worth noting that capacity and volume can sometimes be confused by learners and time should be taken to explore the differences.
Area	The measure of the flat space enclosed by a given boundary. Area is measured in square centimetres (cm^2) and square metres (m^2).
Perimeter	The distance all the way around the outside of a 2D shape. To find the perimeter of a shape, add together the lengths of all the sides. The total is the perimeter.

Definitions for further vocabulary related to measurement can be found in [this glossary].

Early Level

The table below includes the experiences and outcomes related to 'Measurement' at early level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	Benchmarks
I have experimented with everyday items as units of measure to investigate and compare sizes and amounts in my environment, sharing my findings with others. **MNU 0-11a**	• Shares relevant experiences in which measurements of lengths, heights, mass and capacities are used, for example, in baking. • Describes common objects using appropriate measurement language, including tall, heavy and empty. • Compares and describes lengths, heights, mass and capacities using everyday language, including longer, shorter, taller, heavier, lighter, more and less. • Estimates, then measures, the length, height, mass and capacity of familiar objects using a range of appropriate non-standard units.

Establishing Strong Foundations for Learning

Careful consideration should be given to the **spaces**, **interactions** and **experiences** we provide, ensuring that opportunities for learners to develop the concept of measurement permeate across all.

Spaces

Rich, carefully considered learning spaces both outdoor and indoor can offer learners practical opportunities to develop the concept of measurement. Spaces should be planned with a range of objects and materials which encourage learners to explore and investigate measure through their play. Selecting appropriate and engaging resources can enhance interactions, leading to creativity, curiosity and deeper learning.

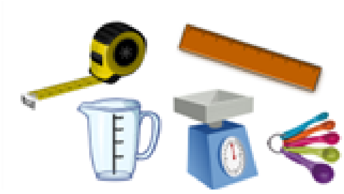
Open-ended materials offer the potential for creative explorations through child-initiated and adult initiated learning experiences. Practitioners should ensure that planning for learning starts with the learner and is carefully balanced to be both responsive and intentional in design. Opportunities should enable learners to make sense of measurement in the world around them, whilst also ensuring learners' needs are being met through their engagement with the experiences and outcomes presented within early level.

There are many ways of permeating measurement across the learning spaces. Learners need to be able to easily access materials that can be used in their play. Some examples of resources are provided below. These items could also be used to create tinkering areas or tables. [2]

Empty containers of varying size and shape

Variety of non-standard measuring tools

Variety of standard measuring tools



Two pan balance scales

Malleable dough and different lengths of string, ribbon, etc

Items of varying length, mass and size

Pourable items

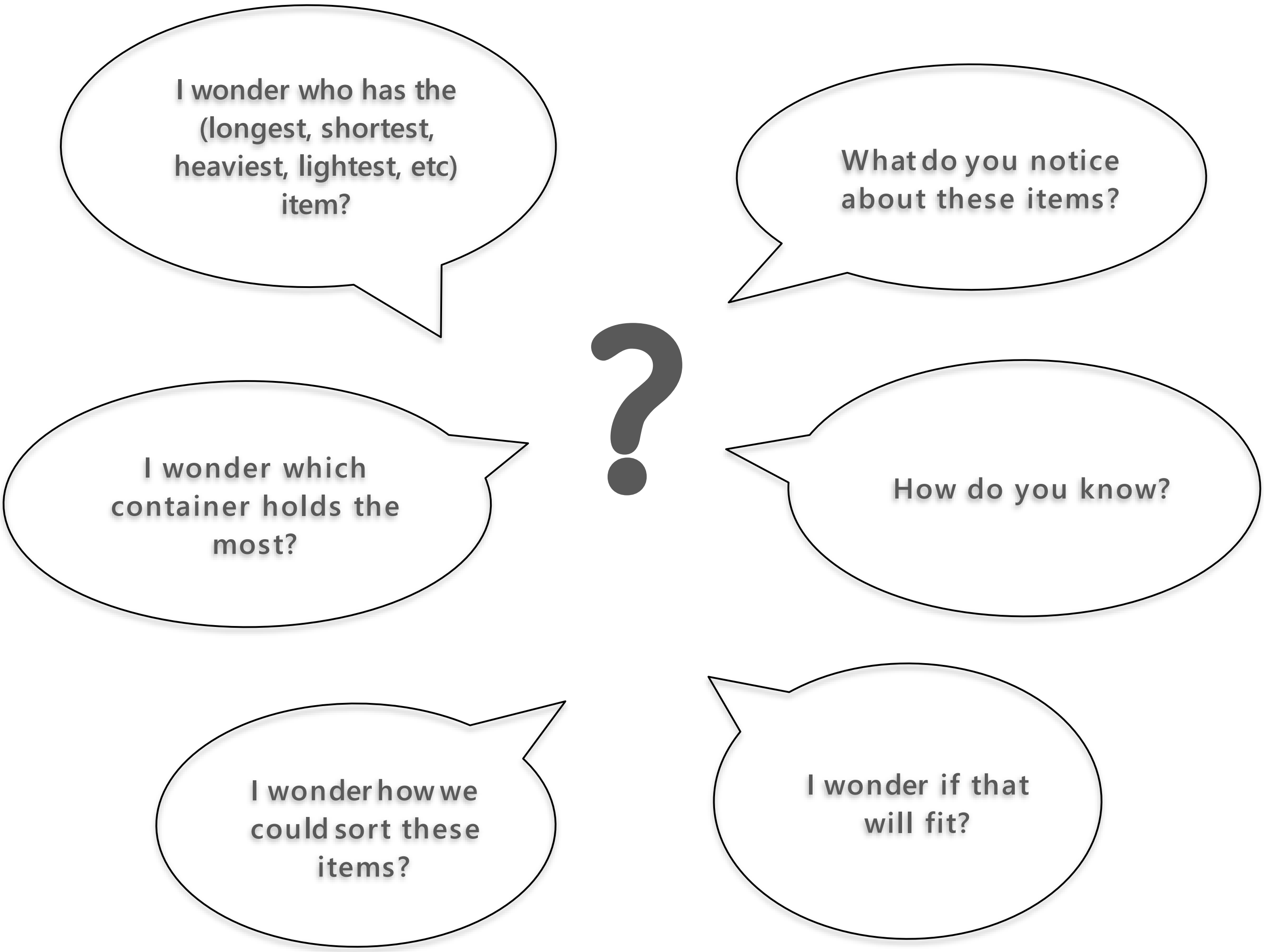
Variety of funnels, scoops, ladles, spoons

[2] Some of the images within this document have been taken from https://pixabay.com/

Interactions – One of the Roles of the Practitioner

One of the roles of the practitioner is to determine what the child could learn through their own interests using high quality interactions. Practitioners should support learners to extend their understanding of measurement through encouragement of creativity and curiosity. Careful observation is an important assessment tool, as is knowing when to stand back and give learners time to investigate by themselves. It is important to notice when learners are more receptive to further support from the practitioner and address any misconceptions that may cause challenges at later stages.

Language can be a powerful vehicle for the teaching and understanding of measurement. When interacting with learners, practitioners can model the use of relevant vocabulary in meaningful contexts. This allows learners to make links between the spoken word and the associated learning. Using language in the correct context regularly will support learners' understanding.



Experiences and Routines

Experiences of everyday activities are essential, both indoor and outdoor. An early appreciation of the contexts in which measurement is used is important. A range of possible experiences are explored over the next few pages.

Role Play and Real Life Situations

Measurement skills are used in many different life and work contexts. This can provide opportunities for a wide variety of role play experiences where measurement skills could be used. It is important that experiences are led by the learners as often as possible.

There are many types of shops or businesses which could be set up, for example, a shoe shop, a supermarket, a post office, a bakery, a restaurant, a DIY shop, an airport or anything that learners might demonstrate an interest in or that is relevant to their local context and environment. Learners could measure the lengths of feet for shoes, weigh parcels and measure their size, measure out ingredients, work out how heavy or light the suitcases are at the airport or plan out interior/exterior layouts.

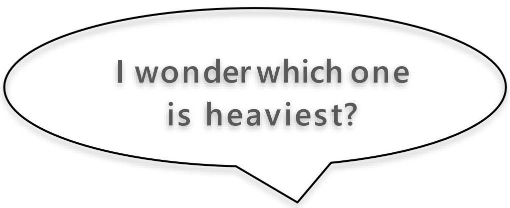
Learners could also be provided with opportunities to visit local shops or businesses to experience how measurement is used in real life situations.

Tinker tables or tinker areas can be a good way for learners to explore and investigate measurement and can lead to powerful provocations built on their interests. This could be followed by up a book and/or suitable activities to extend further and can allow practitioners to observe particular interests and identify potential misconceptions/misunderstandings. Ideas for items to include on tinker tables can be found on page 6 of this document.

When rearranging elements of your setting, whether indoors or outdoors, learners could be involved in the planning of where items should go. Rich dialogue in relation to what might be 'too big' or what might 'fit well' and why could take place.

Estimating

Learners could be encouraged to make links between their estimation and measuring skills.



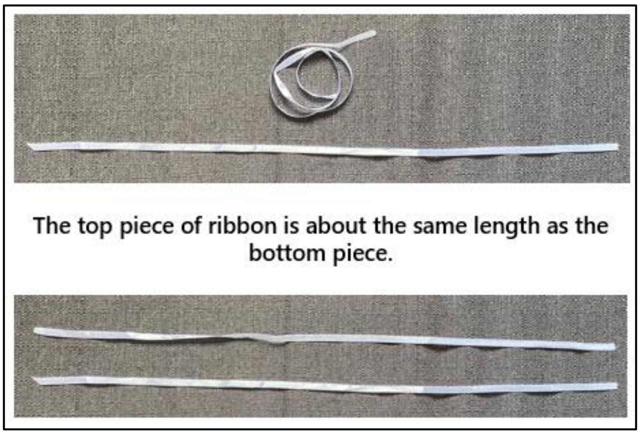
To begin with, this might be making estimates about which item is the longest/shortest, lightest/heaviest or holds most/holds least. This then might progress to 'I think it will measure the same length as five books', ' or 'I think it will take ten cups to fill that bucket.'

Estimations made by learners may be quite far off the actual measurement to begin with, however, the more opportunities they have to estimate and then check by measuring, the more accurate these should become.

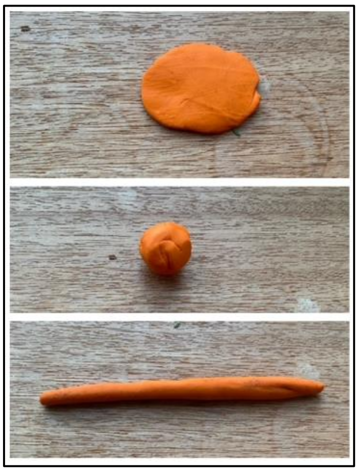
Conservation of Measurement

Opportunities to develop a conservation of measurement, which is an understanding that an object still has the same length, mass or capacity even when the appearance has changed, are very important.

Learners could be provided with opportunities to explore:
- a length of rope, ribbon or string remaining the same length whether it is rolled up or laid out straight
- liquid being poured from a tall and thin container and remaining the same amount when poured into a shorter and wider container
- a ball of modelling clay remaining the same mass whether it has been squashed or rolled.



The top piece of ribbon is about the same length as the bottom piece.



Non-Standard Units

It is important to ensure that there are a range of non-standard units available for learners to access across your setting. These are items which can vary in shape, size and weight, hence the reason they are referred to as non-standard. Learners can be encouraged to measure different objects, spaces and items using these.

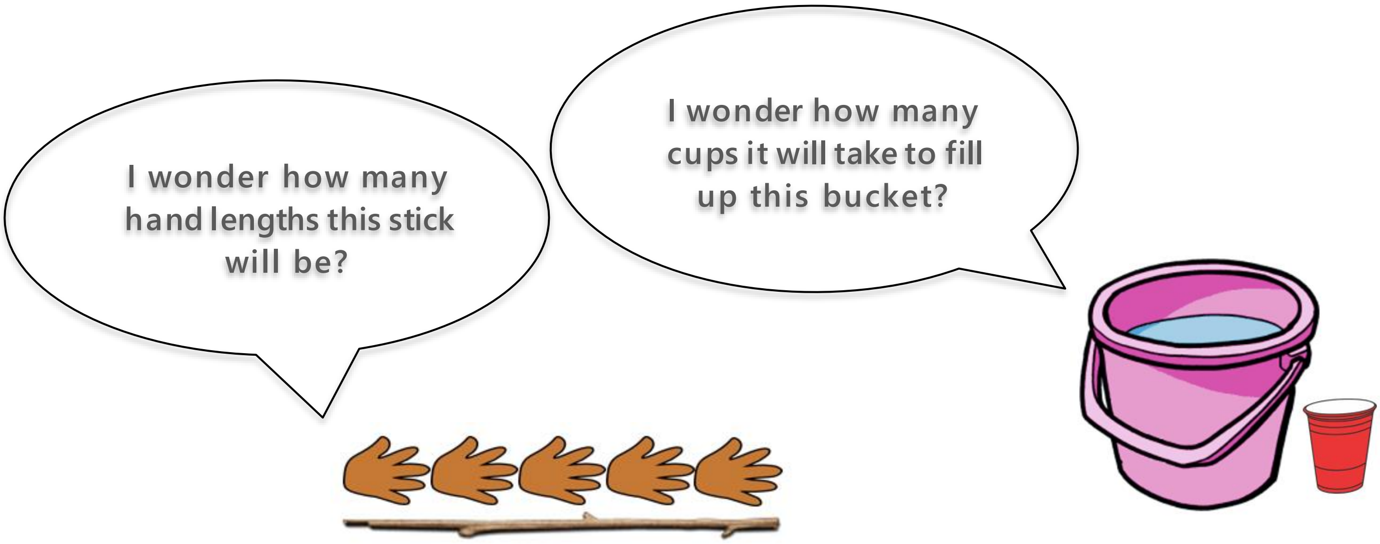
Examples of non-standard units

- Body parts, for example, hands, feet, whole body, etc
- Pencils, paperclips, books, cubes, counters, etc
- Sticks, leaves, pebbles, etc
- Toy cars, small world people or anything of interest!

Examples of standard units

- Millimetres, centimetres, metres, kilometres, etc
- Grams, kilograms, etc
- Millilitres, litres, etc

Using non-standard units can be beneficial to begin with as they can be items which are already familiar to learners. Additionally, using non-standard units can allow the practitioner to control the range of numbers that learners will be working with. For example, using larger items to measure will likely require less complex counting.

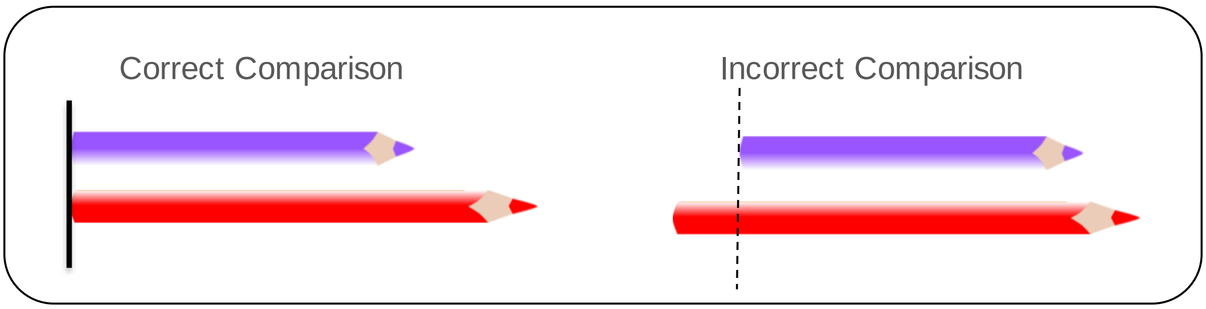


Formal measuring tools such as rulers, scales and measuring jugs should also be made accessible to learners at early level. Having access to these measuring tools within your setting enables learners to explore and investigate how these are used and builds up familiarity with the measuring instruments. It is important to note that learners are not expected to read the scale of measuring tools until they are working within first level.

Making Comparisons

To begin with, learners should be provided with opportunities to compare the attributes of two or more objects to establish which item is longer/shorter, heavier/lighter, holds more/holds less.

Comparing the height of vertical items, such as models they have built or flowers they have grown can be easier than comparing the length of items which have been laid out horizontally. This is because the vertical items will most often already begin at the same starting point, the floor or a table, whereas when items are laid horizontally it is important to ensure that each item starts at the same point and is level with each other.



Making comparisons in relation to mass and capacity can be challenging as often the size and shape of an object or a container can be misleading. Learners might believe that a tall, narrow container always holds more liquid than a short, wide container or that a larger object is always heavier than a smaller object. This misconception should be challenged and explored with learners.

Examples of other direct comparisons learners can make:

Comparing Length/Height

The following objects could be compared to see which are the longest, shortest, tallest, etc:

- sticks or leaves that have been collected
- models created by the learners using different sizes of blocks or boxes
- things they have grown such as sunflowers, cress or plants
- lengths of ribbon or string
- small world characters or other toys
- anything that your learners find and can compare.

Comparing Capacity

Learners could make comparisons using containers of exactly the same size and shape, for example, two cups which are the same size or two buckets which are the same size, etc. Each container can be filled with different amounts of water and then directly compared.

Tip – using food colouring and clear containers can make the liquid easier to see.

Comparing Mass

To begin with, learners can directly compare the mass of two items using their hands, by attempting to feel which is the heaviest and the lightest. This can then progress to the use of a two pan balance to make comparisons. Some items that they could compare might include:

- items of food, for example, fruit, vegetables, tins, bags of shopping, etc.
- items collected outdoors, for example, sticks, stones, feathers, leaves, etc.
- toys and objects found within the setting
- buckets of sand, mud or water
- wrapped parcels of different shapes and sizes
- anything else that your learners would like to compare.

It is worth taking time to explore with learners that when one side of a two pan balance goes down it contains the heavier object and vice versa for the side that goes up.

As learners progress they should be provided with opportunities to make indirect comparisons. Often these are objects which cannot be easily compared side by side, for example, items which are not easy to move around or are different sizes and shapes. Learners can begin to use non-standard units to make comparisons. Some examples are noted below.

Capacity

Learners often enjoying taking part in filling and pouring experiences and providing a range of containers and pourable material will support them to develop their understanding of capacity.

Learners might use different containers, for example a bucket and a jug and investigate and compare how many cups of water it takes to fill up each container. Remember to model and encourage learners to make estimates.

Mass

Learners pick two items of their choice. Place the first item on one side of a two pan balance and see how many crayons it takes to bring the balance back to level. Repeat with the second item they have chosen. Take time to discuss which item is heavier and how they know.

NB. Crayons can be substituted for any other suitable items, e.g. pennies, cubes, etc.

Length

- Use footsteps to measure the length of a rug and compare it to the space inside the role play area to see if it will fit.
- Use hand spans to measure the width of a table to check if it will fit through the door.
- Use interlocking cubes to measure if the tablet will fit in the tray.

The tablet measures about the same as 12 cubes.

There are many other comparisons learners can make; practitioners should follow their interests and let them decide what they would like to directly compare. It is also important to be sensitive when making comparisons and it is often better to use objects and spaces rather than making comparisons between the learners themselves.

Further Potential Experiences

In this section there are some further examples of measuring experiences that might occur. Experiences are not limited to these examples, there are many more opportunities that may occur. It is important to follow the lead and interests of your learners.

Malleable Dough

Use malleable dough to create snakes or worms. Compare the lengths of the creations. Extend this further –

- 'I wonder if we could make a longer worm?'
- 'I wonder how we could make these an equal length?'

Scavenger Hunt

Challenge learners to find the following items:

- A leaf that is longer than their thumb.
- A stick which is about the same length as their arm.
- A stone which is bigger than their thumbnail.

(These items can be substituted for anything else appropriate).

Rock Hunt

Go outdoors and challenge learners to search for two rocks or large stones.

- Which one feels heaviest?
- How can we check?
- Can we find an item that is heavier/lighter than this?

Which is the Heaviest?

Fill or wrap a variety of boxes, ensuring that each box has items of varying mass in them, e.g. a brick, a small ball, etc. Ask learners to pick up the parcels and estimate which one will be heaviest/lightest. Check using a two pan balance.

Bake a Cake

A standard sponge cake requires the flour, eggs and sugar to all have the same weight. Learners could use two pan balances to measure out the correct sugar and flour by using two eggs as a guide.

Although learners are not expected to read scales at this stage, they can still take part in other baking experiences supported by an adult. This will provide them with opportunities to experience measuring in real life situations.

How far will it go?

A range of sizes of water squirters or spray bottles could be made available.

I wonder which one will squirt water the furthest?

What do you notice?

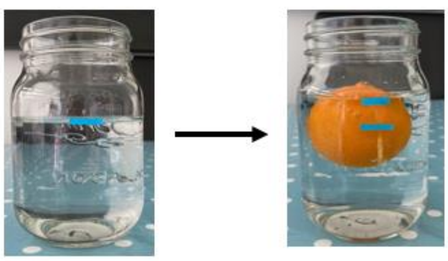
Fill It Up

Use two large, identically sized containers, such as two buckets, alongside smaller containers of varying capacities such as jugs, bottles or cups. Ask learners to fill up each bucket to the top using one of the smaller containers. They should begin to notice that the container with the largest capacity fills the bucket in less pours.

I wonder if it would be easier to fill the bucket up with the bottle or the cup?

Displacement

Fill a container with water, e.g. a jar or a bucket and mark the water line. You could wonder out loud about what might happen to the water when you drop something in it. Try it out with objects of different size and mass and check the water line each time.



What should we use?

Take time to discuss appropriate ways of storing or using items within your setting:

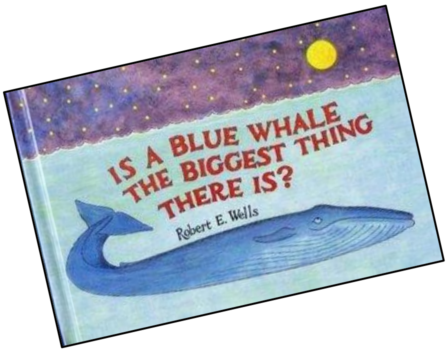
- Which bag/box would this fit in?
- Where could we store this (large) item?
- Which container should we use?
- How can we fill up the water tray quickly?

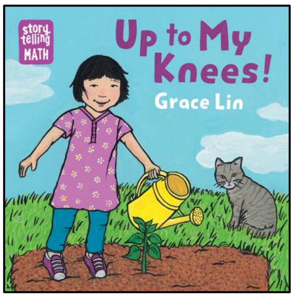
Ask learners to justify their thinking.

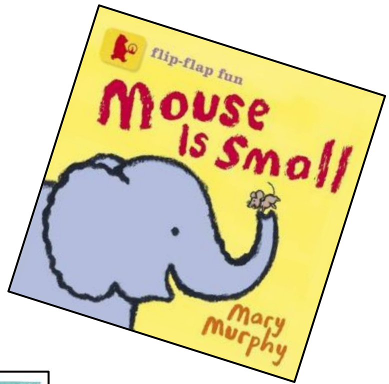
Measurement Stories

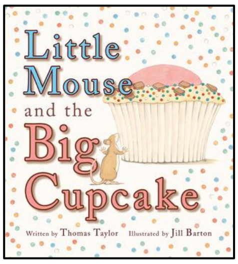
In Early Learning and Childcare and early primary we know that stories can connect young learners to many aspects of learning and development through diverse, meaningful and rich contexts. Their words and rhythms provide a fun and creative way of exploring numeracy concepts alongside developing associated vocabulary. Our brains have become hardwired to respond to stories and learners can use them to make sense of the world. Repetition reinforces learning through encouraging a young child's natural [schematic](#) behaviour.

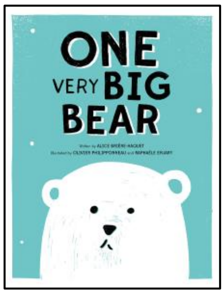
Through a range of traditional and contemporary stories we can engage learners' interests and curiosity and help them understand abstract concepts. There are many stories that relate to measurement, some are listed below[3] but there are many more available.











There are also some traditional tales such as; Goldilocks and the Three Bears, Jack and the Beanstalk and Three Billy Goats Gruff, which can promote discussion in relation to measurement.

Points to consider:

- Learners may believe that the taller the container, the greater capacity.
- The size of an object is not necessarily linked to how heavy it is. Larger objects can be lighter than smaller objects, depending what they are comparing.
- When comparing lengths, ensure learners know to position objects at the same starting point.

[3] These images have been taken from www.goodreads.com

Reflective questions:

- **In what ways can we use our spaces, interactions and experiences to embed the concept of measurement within our setting?**
- **How can we build upon the interests and choices of our learners to incorporate measure in a meaningful way?**

Links across the curriculum:

Numeracy and Mathematics

- Number
    - i. counting
    - ii. addition and subtraction within 10

- Estimation – estimation in the context of measure

- Properties of shape – straight, round, curved, etc

- Data and Analysis – uses knowledge of size and other properties to sort items

Technologies

- Food technology – exploring different types of food

Science

- Inquiry & investigative skills
    - i. makes simple predictions
    - ii. measures using simple equipment and non-standard units

First Level

The table below includes the experiences and outcomes related to 'Measurement' at first level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	Benchmarks
I can estimate how long or heavy an object is, or what amount it holds, using everyday things as a guide, then measure or weigh it using appropriate instruments and units. **MNU 1-11a** I can estimate the area of a shape by counting squares or other methods. **MNU 1-11b**	• Uses knowledge of everyday objects to provide reasonable estimates of length, height, mass and capacity. • Makes accurate use of a range of instruments including rulers, metre sticks, digital scales and measuring jugs when measuring lengths, heights, mass and capacities using the most appropriate instrument for the task. • Records measurements of length, height, mass and capacity to the nearest standard unit, for example, millimetres (mm), centimetres (cm), grams (g), kilograms (kg), millilitres (ml), litres (l). • Compares measures with estimates. • Uses knowledge of relationships between units of measure to make simple conversions, for example, 1 m 58 cm = 158 cm. • Reads a variety of scales on measuring devices including those with simple fractions, for example, 1/2 litre. • Uses square grids to estimate then measure the areas of a variety of simple 2D shapes to the nearest half square. • Creates shapes with a given area to the nearest half square using square tiles or grids. • Recognises that different shapes can have the same area (conservation of area).

Effective learning and teaching approaches

Estimating

Before carrying out any type of measurement activity, learners should first be asked to make an estimate of what the item they are measuring might be. To begin with this might follow on from early level where learners made estimates about which item is the longest/shortest, lightest/heaviest or holds most/holds least. It can also follow on from estimating how many non-standard units (see page 8) were used to measure a particular item. Learners should always be asked to explain their thinking and then be given time to test it out. Estimations made by learners will likely be quite far off the actual measurement to begin with, however, the more opportunities they have to estimate and then check by measuring, the more accurate they should become. Time to estimate should be provided for all measurement experiences.

Before moving onto using standard units learners should estimate the measurement of a variety of everyday items by making comparisons. For example, they could hold a pencil case in their hand and try to find something around them which is heavier than that. This could then be checked using a two pan balance to check which of the items is the heaviest. Further examples are provided below:

Find something heavier/lighter than:

- your schoolbag
- this stick
- your shoe
- anything else of interest.

Find something longer than/shorter than:

- your hand
- a leaf
- your pencil
- anything else of interest.

Items could be directly compared to check if the estimation was correct.

Standard Units

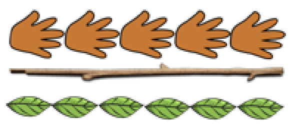
At first level learners will begin to use standard units such as kilograms, grams, metres, centimetres, litres and millilitres. It can be beneficial to revisit non-standard units to begin with. Measuring and estimating with non-standard units can support understanding of why standard units are necessary.

Why is it different?

Provide learners with opportunities to measure the same item using different non-standard units, e.g. shoes, pencils, etc. Discuss why the measurement is different each time (because each time we have used things that are different in size). At this point a standard measurement could be introduced, e.g. one metre, one kilogram, one litre.

It takes about 10 bottles to fill up this bucket with water.

It takes about 26 cups to fill up this bucket with water.



This stick is about the same length as 5 hands.

This stick is about the same length as 6 leaves.

To begin with learners could explore what one metre (m), one kilogram (kg) and one litre (l) looks and feels like. Learners could use both indoors and outdoors to find a range of different items. **Units of measurement should be introduced one at a time.**

Find something that is:

- about the same length as one metre
- about one kilogram
- about one litre

Find something that is :

- shorter/longer than one metre
- more than/less than one kilogram
- more than/less than one litre

Can you find two sticks which measure about the same as one metre?

How many stones can you find that would measure about one kg?

How many full cups will it take to fill up this one litre bottle?

When introducing litres, learners should be provided with a variety of shapes and sizes of containers that hold approximately one litre and be given opportunities to explore the fact that although the containers look different, they actually hold the same amount of liquid. Similarly, when introducing kilograms, opportunities to measure out one kilogram of different items can help learners understand that one kilogram will not always look the same, even though the mass remains the same.

1 litre of liquid in each different container

1 kg of rice 1 kg of bananas

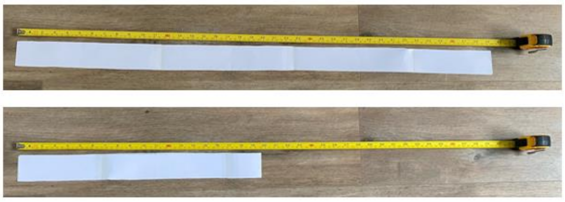
Mass is a measure of how much matter there is in an object and is often measured using grams and kilograms while weight is a measure of the size of the pull of gravity on the object and is measured in newtons. This can be confusing for learners as many everyday experiences talk about 'weighing' – weighing the fruit, weighing the flour. However mathematically we are actually determining the mass of these items. You could explore this concept further with learners by explaining that the mass of an object will always remain the same but the weight of an object changes depending on the pull of gravity. For example, an astronaut might weigh 560 newtons on Earth but only 90 newtons on the moon. However, the astronaut would always have a mass of 82 kg.

As learners progress through first level, similar measurement experiences could be provided in relation to half and quarter metres, kilograms and litres. The link between the whole, the half and the quarter should also be explored, for example, two halves are equal to one whole, two quarters are equal to one half and four quarters are equal to one whole. This will provide opportunities for learners to consolidate and extend their understanding of fractions and measurement. The ideas below provide some examples for experiences that could support learner understanding of half quantities, however, these could also be used when exploring quarters too.

Half a Metre

Learners create their own paper metre strip. Fold the strip in half. Now we have half a metre (m). Find items that are:

- about half a metre
- shorter than half a metre
- longer than half a metre



Half a Kilogram

Use a 2 pan balance to measure out one kilogram (kg) of malleable dough. Split the dough in half across the two pans, levelling the balance to ensure each part of dough is equal. Remove one half of the dough. Now find items that are:

- about half a kg
- more than half a kg
- less than half a kg

In addition to dough, learners could also measure out 1kg of rice, sand, marbles or anything else that can be easily split into halves.

Half a Litre

Provide learners with a container which has capacity of one litre and another which has capacity of half a litre. Learners could be given time to pour liquid from container to container, what do they notice? How many of the smaller containers does it take to fill up the larger container?

Formal measuring equipment can then be introduced where one litre, half a litre and a quarter of a litre are marked e.g. a measuring jug or cylinder.

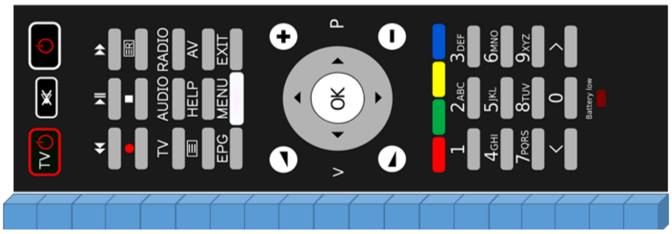
Once learners have had a wide range of practical opportunities to work with full, half and quarter measurements of metres, kilograms and litres they can begin to explore smaller measurements such as centimetres (cm), grams (g) and millilitres (ml).

Introducing Centimetres

Discuss items or objects that we have previously measured using metres, for example, length of classroom, height of the door, length of the adventure play structure, etc.

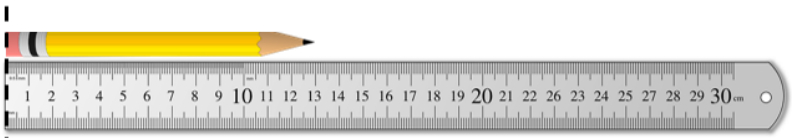
Provide learners with a range of smaller items that would be too difficult to measure using a metre stick. Ask learners why it might be challenging to use the metre stick to measure these items. Following this discussion the centimetre could be introduced as a standard unit of measure that could be used to find the length of some of the smaller items we would like to measure.

Before introducing a ruler to measure, centimetre cubes could be lined up beside an item and counted.



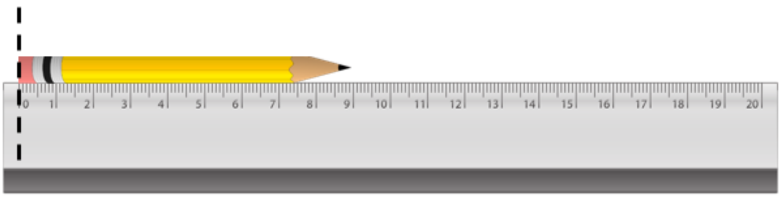
When you are ready to introduce the ruler, it is important that time is taken to investigate how to use a ruler correctly. This will likely involve a great deal of modelling and practical exploration. The rulers that will be available to learners will vary. On some rulers (Type A) measuring should begin using the very edge of the ruler and on other rulers (Type B) there is a small gap between the edge of the ruler and where the measuring should begin. It can be beneficial to provide opportunities to use both types of ruler.

Type A



Objects are measured from the edge of the ruler. They must be placed in line with the edge. Often the zero is not marked on these types of rulers.

Type B



Objects should be measured using the zero mark as a starting point. Learners can forget to do this and often start from the edge.

It can also be worth taking time to generate discussion about the correct way to use a ruler.

Three friends are measuring the length of a bank card. Who is the most accurate? Why?

There are many practical experiences that could be provided to support learners to explore centimetres. The activities below could act as a starting point.

Equal Measures

Explore how many centimetres are the same length as one metre, one half of a metre and one quarter of a metre. You could do this by lining up cubes which are 1 cm in length.

Measuring Tools

Explore the different measuring devices that can be used to measure centimetres e.g. rulers, metre sticks with cm markings, measuring tape, etc, Provide learners with opportunities to explore these tools and investigate the benefits of each depending on what is being measured.

Jump

Learners could estimate in centimetres how far they can jump with two feet together. Make the jump and then use a metre stick marked in centimetres to measure. Repeat a few times and the estimates are likely to become more accurate.

Worms

Provide learners with equal sized balls of malleable dough. Challenge them to roll the longest 'worm' they can in 20 seconds. Once complete, measure each worm and see who managed to roll the longest one by measuring each one using centimetres.

Millimetres can be introduced when learners have had plenty of opportunities to explore and work with measurements using centimetres. You could provide learners with something very small such as a grain of rice or an individual staple. Discuss why it might be challenging to use centimetres to measure these items. Following this discussion the millimetre could be introduced as a standard unit of measure that could be used to measure the length of very small items. Some of the previous activities linked to centimetres can be adapted when working with millimetres.

Who Uses Millimetres?

Explore some of the jobs which require measurements to be taken using millimetres. Some examples are below but there are many others that could be explored:

joiners architects

dentists

plumbers jewellers

How Many?

Explore how many millimetres are the same length as one centimetre, two centimetres, ten centimetres, etc.

What should we use?

Discuss with your learners, the most appropriate tool to measure the length, width or height of different lengths e.g. what unit of measurement should we use to measure:

- the width of the door?
- the length of the corridor?
- the length of a pencil?
- the height of the window?
- the width of your jotter?
- the length of a staple?

Introducing Millilitres

Millilitres

Discuss items or objects that we have previously measured using litres, for example, bottles of milk, juice, buckets of water, etc.

Provide learners with a range of smaller items that would be too difficult to measure using litres e.g. a small glass of water, liquid on a teaspoon, etc. Ask learners why it might be challenging to use litres to measure these items. At this point the millilitre could be introduced as a standard unit that could be used to measure the capacity of smaller items.

Take time to discuss and explore which containers are best measured using litres and which ones with millilitres. Look for containers at school and at home which are measured using millilitres.

Millilitres

Explore the different measuring devices that can be used to measure millilitres e.g. measuring jugs, cylinders, syringes, measuring cups, etc. Provide learners with opportunities to explore using these devices with large buckets of water.

Puddles

Estimate how many millilitres a puddle outside measures. Use plastic syringes to collect the water and then measure in a large measuring cylinder.

Turn it into a challenge – which group can find the puddle which holds the most amount of water?

Millilitres

Provide learners with a range of sizes of containers, each one filled with a different amount of liquid. Learners should estimate and place the containers into order from what they think 'holds the least' to 'holds the most'. The liquid from each container should then be measured using a measuring cylinder marked in millilitres. Were there any surprises? If so, why might this be?

Introducing Grams

Grams

Discuss items or objects that we have previously measured using kilograms, for example, animals, potatoes, bags of flour, etc.

Provide learners with a range of smaller items that would be too difficult to measure using a kilogram e.g. a pencil, a piece of jewellery etc. Ask learners why it might be challenging to use kilograms to measure these items. Following this discussion the gram could be introduced as a standard unit of measure that could be used to find the mass of some of the smaller items we would like to measure.

Find items around the classroom and outdoors that would be appropriate to measure in grams. Estimate how many grams they might be and then check.

Packaging

Provide learners with a range of items of different sizes and shape. Ask them to put them in order of size from what they estimate is the 'heaviest' to 'lightest'. Use the scales to check and rearrange where required.

Are there any surprises? If so, why might that be?

Guess the Label

Provide learners with a range of packaged food items with the mass covered up. Create cards with the correct mass for each item and ask learners to match each item to a card. Use scales to check.

Different Tools

Explore the different measuring devices that can be used to measure grams:

- two pan balance with 500g, 250g, 100g, 50g, 20g, 10g, 5g and 1g weights
- kitchen scales – both digital and mechanical
- Spring/hanging scales, etc

Provide learners with opportunities to explore these devices using a wide range of objects from both indoors and outdoors.

Although our official measuring system is the metric system, it is likely that learners will be aware of measurements that use the imperial system, for example, miles for measuring distance or the use of ounces and pounds when baking. Imperial measurements are formally acknowledged in second level but it may be appropriate to have some discussion about the imperial system depending on the experiences of our individual learners.

Choosing Units of Measurement

It is important that learners are provided with opportunities to make choices about which units of measurement to use. Initially the choices will begin with just a few and as learners progress, the choices can be expanded. To begin with learners may not choose the most appropriate measuring tool or unit of measurement but it is important to let them try it out and reflect on whether there is a more appropriate alternative.

Making Conversions

At first level learners should begin to make simple conversions between units of measurement.

- metres → centimetres
- centimetres → metres
- centimetres → millimetres
- millimetres → centimetres

- litres → millilitres
- millilitres → litres

- kilograms → grams
- grams → kilograms

It can be a good idea to start with very simple conversions such as the ones outlined on the following page and use practical experiences alongside. All conversions should be carried out both ways, for example, 1m = 100cm and 100cm = 1m.

1 m = 100 cm	1 ℓ = 1000 ml	1 kg = 1000 g	1 cm = 10 mm
$\frac{1}{2}$ m = 50 cm	$\frac{1}{2}$ ℓ = 500 ml	$\frac{1}{2}$ kg = 500 g	$\frac{1}{2}$ cm = 5 mm
$\frac{1}{4}$ m = 25 cm	$\frac{1}{4}$ ℓ = 250 ml	$\frac{1}{4}$ kg = 250 g	
$\frac{3}{4}$ m = 75 cm	$\frac{3}{4}$ ℓ = 750 ml	$\frac{3}{4}$ kg = 750 g	

This can then be extended to link with simple addition/multiplication. It is also important to make explicit links to learners' knowledge of doubles and halves.

2 m = 200 cm	2 m = 200 cm	2 cm = 20 mm
3 m = 300 cm	$3\frac{1}{2}$ m = 350 cm	3 cm = 30 mm
4 m = 400 cm	$4\frac{1}{4}$ m = 425 cm	5 cm = 50 mm
8 m = 800 cm	$1\frac{3}{4}$ m = 175 cm	10 cm = 100 mm

Slightly more challenging conversions can then be introduced. Again, link with practical examples as often as possible and ensure that learners are provided with opportunities to convert both ways. It is also important that links are made to place value rather than learning 'tricks' to make quick conversions as this is more likely to lead to a deeper understanding.

3 m 58 cm = 300 cm + 50 cm + 8 cm = 358 cm

273 cm = 200 cm + 70 cm + 3 cm = 2 m 73 cm

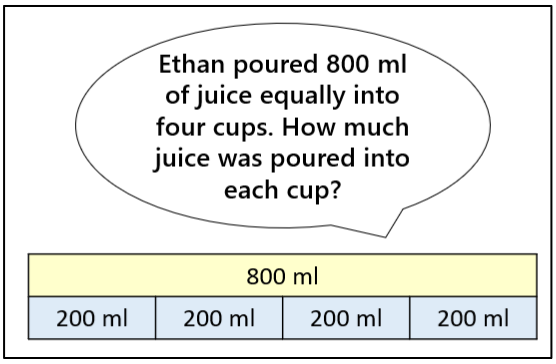
Bar models and cherry diagrams can support understanding of the links between measurement conversions through a visual approach.

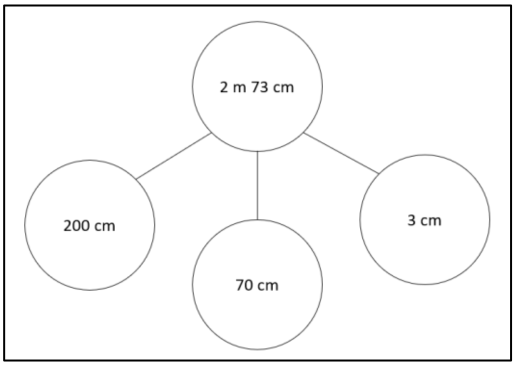
1 kg			
$\frac{1}{2}$ kg		$\frac{1}{2}$ kg	
$\frac{1}{4}$ kg	$\frac{1}{4}$ kg	$\frac{1}{4}$ kg	$\frac{1}{4}$ kg

1 m			
$\frac{1}{2}$ m		$\frac{1}{2}$ m	
$\frac{1}{4}$ m	$\frac{1}{4}$ m	$\frac{1}{4}$ m	$\frac{1}{4}$ m

1000 g									
500 g					500 g				
250 g			250 g		250 g			250 g	
100 g	100 g	100 g	100 g	100 g	100 g	100 g	100 g	100 g	100 g

100 cm									
50 cm					50 cm				
25 cm			25 cm		25 cm			25 cm	
10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm	10 cm

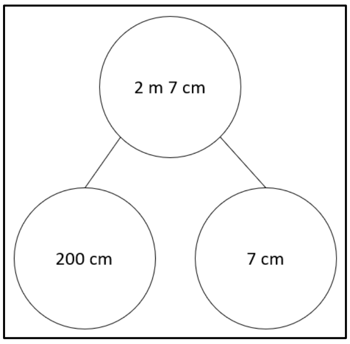


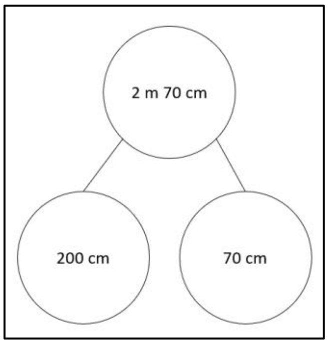


It is important to pay close attention to particular conversions where errors can often be made. This can often arise in conversions where zero is used as a place holder. Further examples of how to build understanding in relation to place value can be found on pages 21 – 28 of the Number and Number Processes Professional Learning Resource.

2 m 7 cm = 200 cm + 7 cm = 207 cm

2 m 70 cm = 200 cm + 70 cm = 270 cm





At first level, learners should not be asked to do conversions which involve decimal fractions. Conversions should be kept simple, similar to the examples provided on this page.

Reading Scales

It is important that time is taken to explore how to read a variety of scales marked with differing intervals and discuss when it is appropriate to use each particular scale. For example, we use scales marked in litres for larger amounts of liquid and scales marked in millilitres for smaller amounts of liquid. In terms of numerical values on scales, learners would be expected to work with numbers within 1000, therefore it is recommended that the intervals on scales used for measuring are marked using numbers which learners are familiar with. Scales using simple fractions such as $\frac{1}{4}\ell$ or $\frac{1}{2}$ kg would be also be appropriate to use at first level. Decimal intervals will be formally explored further in second level.

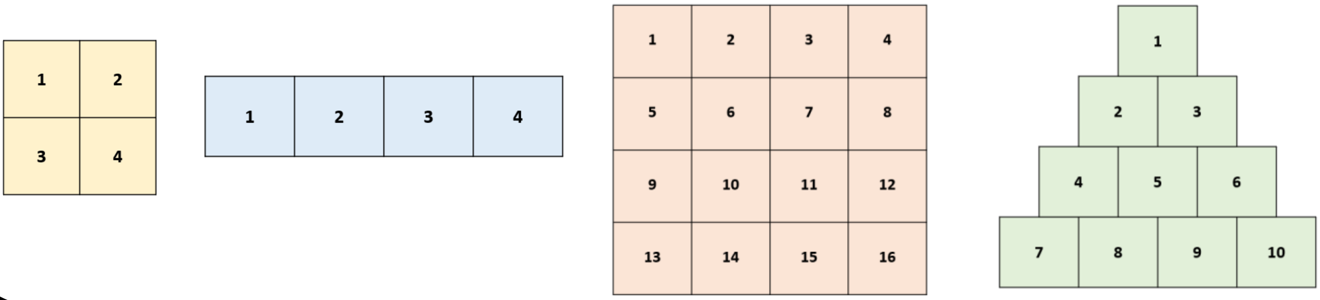
Area

Area is the measure of the flat space enclosed by a given boundary. At first level, area is introduced through counting squares initially and then extending to half squares.

To introduce the concept of ‘area’ learners could be provided with experiences of finding the surface area of regular shapes using non-standard units of measurement. Begin to move language from the space ‘covered’ or ‘taken up’ to referring to the ‘area’.

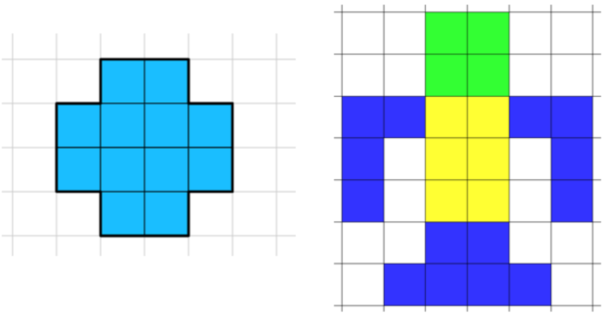
Counting Squares

Provide learners with a range of shapes made up of squares and ask them to count them. You could begin with regular shapes and then progress to counting the squares on irregular shapes.

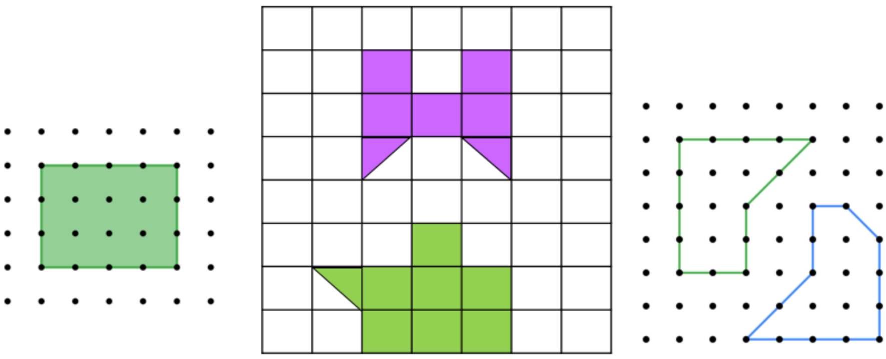


Square Grids

Using squared paper, learners could count the number of squares on each of the different images. Remember to take time to make estimates before counting. Learners could also be challenged to create their own images.

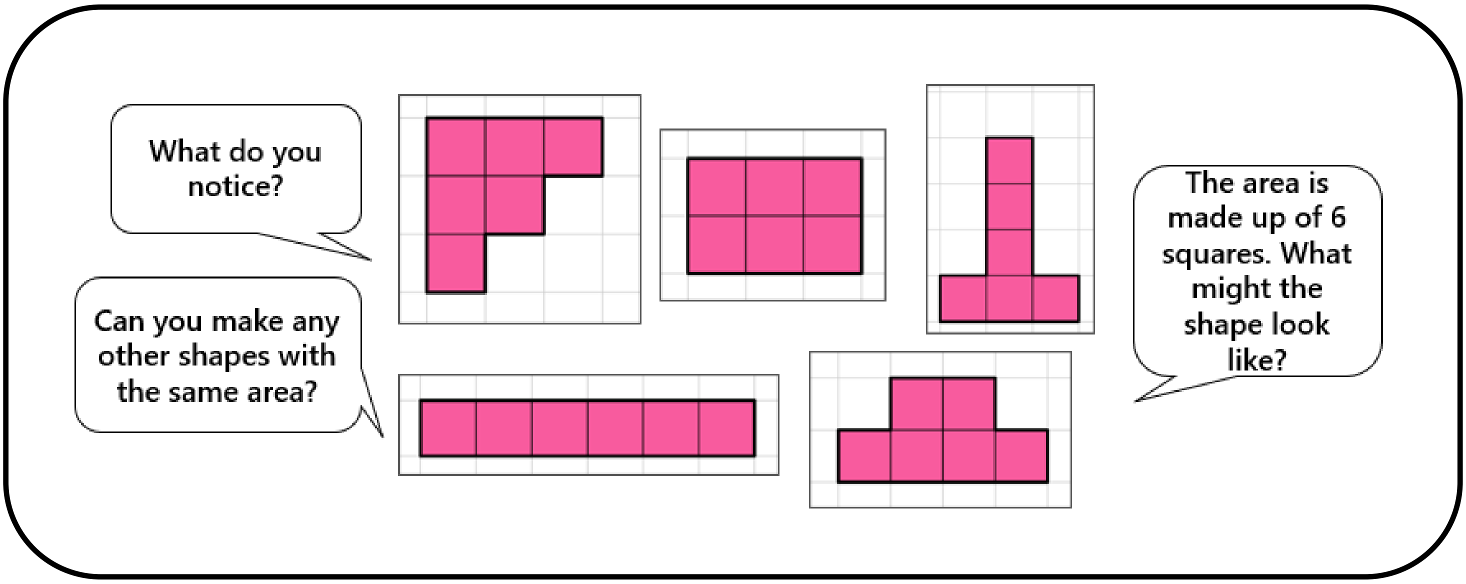


When learners are confident in estimating and counting full squares, half squares could be introduced. Geoboards can also be used to create different shapes with different numbers of squares and half squares.



Digital geoboards can be found at https://toytheater.com/

It is also important to provide learners with opportunities to explore that different shapes can have the same area. This could begin with full squares and extend into including half squares. Learners could be challenged to see how many different shapes they can make with the same number of squares/area.



Points to consider:

- Ensure learners are explicitly taught how to use a ruler to measure accurately and are provided with many opportunities to do so.
- Learners should be encouraged to make estimations before carrying out any type of measuring activity.
- Learners can often forget to include the unit of measurement after the numerical value.
- It is important to reinforce the importance of always noting the unit of measurement after the number, for example, '3 cm' not '3'.

Reflective questions

- **How can we encourage learners to make estimates before they measure items?**
- **How often do we provide opportunities for learners to select the most appropriate tool for measurement?**

Links across the curriculum:

Numeracy and Mathematics

- Number
    i. Whole numbers to 1000
    ii. Place value, including zero as a place holder
    iii. Addition, subtraction, multiplication and division

- Estimation – estimation in the context of measure

- Fractions, decimal fractions and percentages
    i. simple fractions on a number line (scale)

ii. fractions of a quantity

- Expressions and equations – compare measurements using the terms ‘equal to’, ‘less than’ and ‘greater than’.

- Properties of 2D shapes – measuring length, width and area of 2D shapes

Technologies (Food and Textile Technology)

- Use a range of tools and equipment when working with textiles.

Science

- Inquiry & investigative skills
  i. make predictions about the scientific investigations.
  ii. observe and collect information and make measurements using appropriate equipment and units.

Second Level

The table below includes the experiences and outcomes related to ‘Measurement’ at second level. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

Experiences and Outcomes	Benchmarks
I can use my knowledge of the sizes of familiar objects or places to assist me when making an estimate of measure. **MNU 2-11a** I can use the common units of measure, convert between related units of the metric system and carry out calculations when solving problems. **MNU 2-11b** I can explain how different methods can be used to find the perimeter and area of a simple 2D shape or volume of a simple 3D object. **MNU 2-11c**	• Uses the comparative size of familiar objects to make reasonable estimations of length, mass, area and capacity. • Estimates to the nearest appropriate unit, then measures accurately: length, height and distance in millimetres (mm), centimetres (cm), metres (m) and kilometres (km); mass in grams (g) and kilograms (kg); and capacity in millilitres (ml) and litres (l). • Calculates the perimeter of simple straight sided 2D shapes in millimetres (mm), centimetres (cm) and metres (m). • Calculates the area of squares, rectangles and right-angled triangles in square millimetres (mm^2), square centimetres (cm^2) and square metres (m^2). • Calculates the volume of cubes and cuboids in cubic centimetres (cm^3) and cubic metres (m^3). • Converts between common units of measurement using decimal notation, for example, 550 cm = 5·5 m; 3·009 kg = 3009 g. • Chooses the most appropriate measuring device for a given task and carries out the required calculation, recording results in the correct unit. • Reads a variety of scales accurately. • Draws squares and rectangles accurately with a given perimeter or area. • Demonstrates understanding of the conservation of measurement, for example, draw three different rectangles each with an area of 24 cm^2. • Shows awareness of imperial units used in everyday life, for example, miles or stones.

Effective learning and teaching approaches

Estimation

Learners should be encouraged to make estimations before carrying out any type of measuring activity. Estimating can support learners to determine the reasonableness of their solution and can enhance and refine their ‘sense’ of measure.

To begin with, learners should use their knowledge of familiar items to support their estimations. For example;

- using their knowledge of the width of their thumb to make a reasonable estimation of how long a desk is to the nearest centimetre,
- using their knowledge of the mass of a small bag of sugar, in grams, to make an estimation in relation to the mass of a melon,
- using their knowledge of the capacity of a bottle of water, in millilitres, to make an estimation in relation to the capacity of a jug.

Additionally, learners should be provided with opportunities to make estimations using the most appropriate unit of measurement before then going on to check this by using the correct measuring device.

Over time these estimations should become more refined as learners develop a deeper understanding of the different types of measurements and become more familiar with common measurements.

Units of Measurement

Learners should continue to be provided with opportunities to measure a wide range of items using a variety of measuring tools. It is important to give learners the opportunity to select the most appropriate measuring device based on what they are being asked to measure. This could be reinforced by having different measuring devices available for selection. Learners might initially choose the wrong device such as a ruler to measure the length of a large object or a tape measure for something very small. It is important to let the learners attempt to measure with their chosen device then have a discussion about whether there is a better option to try. At second level learners will begin to explore units of measurement with greater values and will also be introduced to kilometres (km).

When introducing kilometres, kilograms can be revisited and close attention paid to the 'kilo' part meaning 1000.

1 kilogram = 1000 grams	1 kilometre = 1000 metres
$\frac{1}{2}$ kg = 500 g	$\frac{1}{2}$ km = 500 m
$\frac{1}{4}$ kg = 250 g	$\frac{1}{4}$ km = 250 m
$\frac{3}{4}$ kg = 750 g	$\frac{3}{4}$ km = 750 m

It can be useful to measure out one kilometre with learners as this can deepen their understanding. This could be carried out in the playground or a larger surrounding area. Additionally, opportunities to explore the distance in kilometres between familiar places, both locally and wider, can support learners to gain a deeper understanding of this unit of measurement. Learners could be timed to see how long it takes them to walk or run one kilometre.

Learners will now have been introduced to millimetres, centimetres, metres and kilometres. They now need to be able to make decisions about which of these units of measurement should be used depending on what is being measured and justify their decision. Providing opportunities to measure a range of items using a range of tools, such as, rulers, measuring tapes, metre sticks and trundle wheels can support learners to develop this skill.

millimetres, centimetres, metres or kilometres? ?

- the length of the school building
- the length of a pen
- the width of a pencil
- the distance between Inverness and Dumfries
- the height of the door
- the length of a train
- the distance between our school and the local secondary school
- the width of a coin

Learners should continue to explore choosing the most appropriate measuring device for length, mass and capacity using all units of measurement. Opportunities to make decisions about the most appropriate unit and measurement tool should be provided alongside time to carry out the measurement and decide how it should be recorded. It is important to note that some measuring scales use the imperial measurement system and whilst learners should develop an awareness of this (see section on imperial measurements, page 51) they are only expected to use the metric system to carry out measurements.

Examples of different measuring equipment:

Measuring Mass

mechanical/analogue scales

digital scales

hanging scales e.g. luggage scales

Measuring Capacity

different sized beakers

measuring jugs

syringes

measuring spoons (marked with ml)

Measuring Length/Distances

rulers

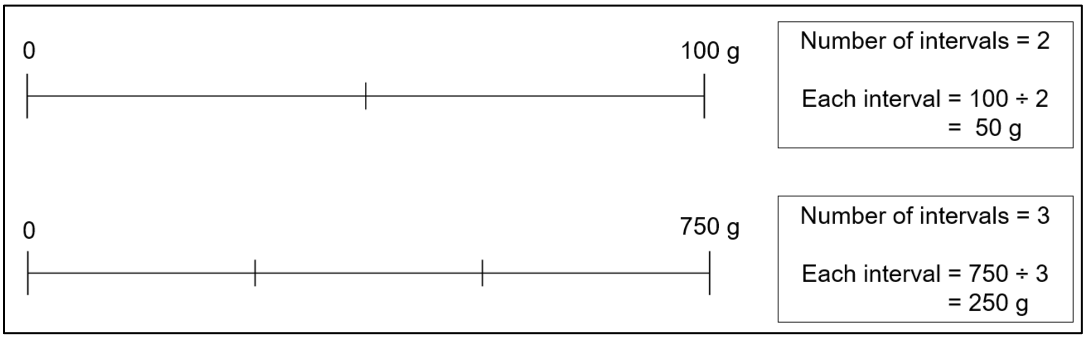
measuring tapes (including those that measure over one metre)

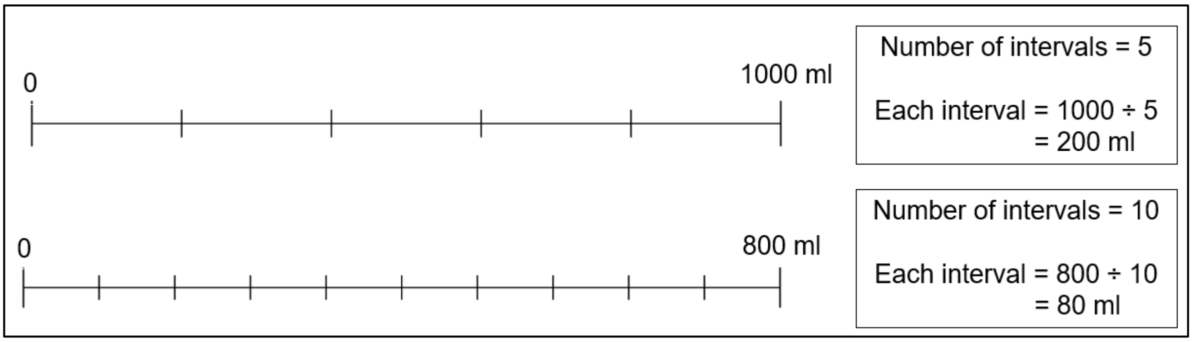
metre sticks

trundle wheels

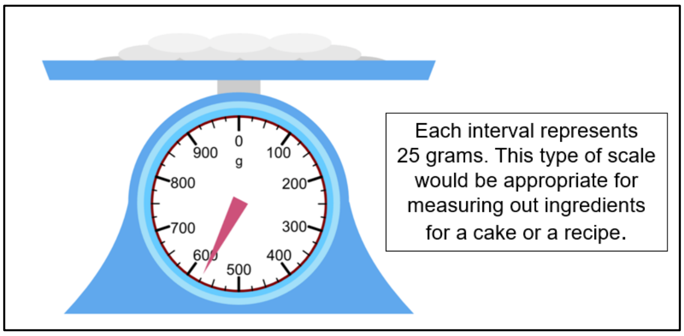
Measuring scales often use differing intervals to record the measurements. The type of intervals used on each piece of equipment will vary depending on the intended use and the level of accuracy that is required.

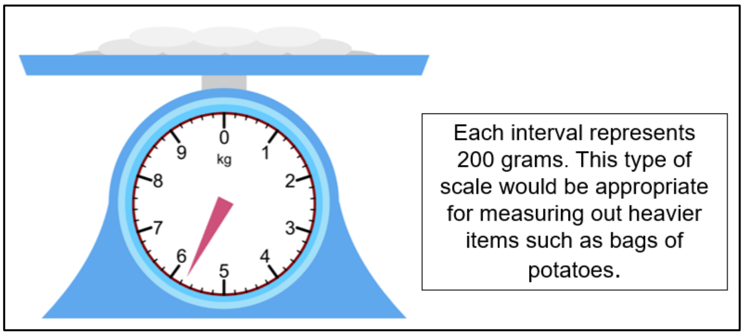
To begin with, learners could use number lines to investigate how to calculate the intervals before moving onto reading them directly from measuring tools. Intervals on all measuring scales should be evenly distributed.





Time should be taken to explore how to read a variety of scales with differing intervals and discuss when it is appropriate to use each particular scale. Some examples of differing intervals and potential discussion prompts are noted below.





I wonder why only some of the intervals have numbers written on them?

Which measuring scale would be most suitable to measure this particular object? Why?

I wonder why some interval lines are larger than others?

Examples of varying intervals on scales used to measure liquids.

Making Conversions

At second level learners should begin to make more challenging conversions within each unit of measurement. Again, this should be linked with practical examples as often as possible and learners should be provided with opportunities to convert both ways. It is also important that links are made to place value rather than learning 'tricks' to make quick conversions as this is more likely to lead to a deeper understanding.

2 km 764 m = 2000 m + 764 m = 2764 m

7582 m = 7000 m + 582 m = 7 km 582 m

4 ℓ 765 ml = 4000 ml + 765 ml = 4765 ml

3972 ml = 3000 ml + 972 ml = 3 ℓ 972 ml

2 kg 549 g = 2000 g + 549 g = 2549 g

4732 g = 4000 g + 732 g = 4 kg 732 g

Making the link between place value and measurement is particularly important when carrying out conversions where zero is used as a place holder. It is important to spend time specifically exploring conversions which involve zero as a place holder.

2 km 7 m = 2000 m + 7 m = 2007 m

4 km 70 m = 4000 m + 70 m = 4070 m

8 km 700 m = 8000 m + 700 m = 8700 m

2 ℓ 5 ml = 2000 ml + 5 ml = 2005 ml

4 ℓ 50 ml = 4000 ml + 50 ml = 4050 ml

3 ℓ 500 ml = 3000 ml + 500 ml = 3500 ml

3 kg 5 g = 3000 g + 5 g = 3005 g

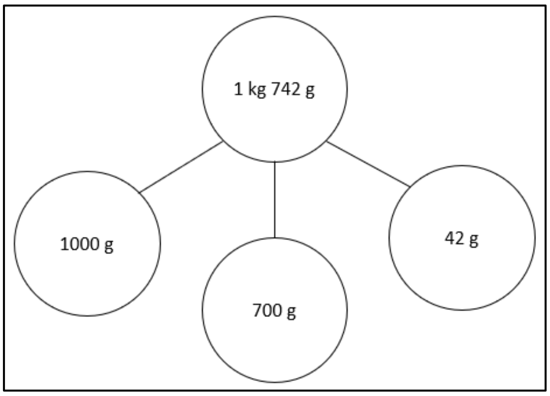
2 kg 50 g = 2000 g + 50 g = 2050 g

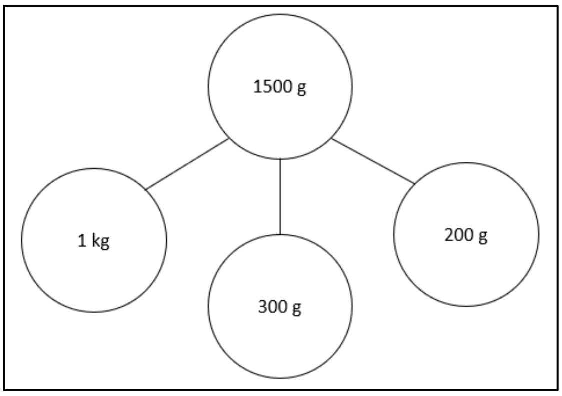
4 kg 500 g = 4000 g + 500 g = 4500 g

Bar models and cherry diagrams can support understanding of the links between measurement conversions through a visual approach.

1500 g		
1 kg	500 g	
1 kg	250 g	250 g
1 kg	400 g	100 g

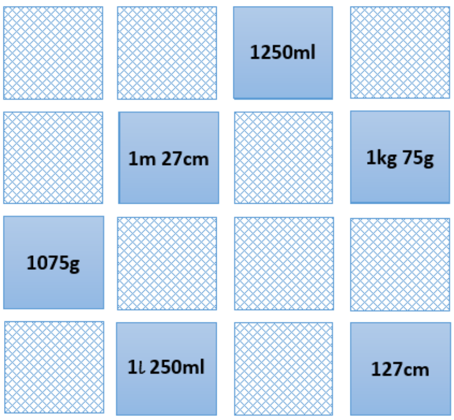
4587 ml						
1 ℓ	1 ℓ	1 ℓ	1 ℓ	500 ml		87 ml
1 ℓ	1 ℓ	1 ℓ	1 ℓ	250 ml	250 ml	87 ml





To reinforce conversions learners could play simple matching games such as ‘snap’ or ‘matching pairs’ to match equivalent measurements.

Matching Pairs



Learners could play matching pairs where they have to take turns of turning over two different cards. If the measurements match they get to keep the pair and continue to take turns until they do not have a match. The winner is the person with the most matched pairs once all cards have been matched.

N.B. This game shows mixed measurements. If focusing on one particular type of measurement e.g. capacity, it might be better to stick to just litres and millilitres.

Greater Than

Provide learners with two piles of measurement cards of one type of measurement e.g. mass. One pile of cards is made up of varied measurements written in ‘grams’ only e.g. 3723 g and the other pile is made up of measurements written in ‘kilograms and grams’ e.g. 5 kg 67 g.

Learners draw a card from each pile and then use the ‘greater than sign’ to place the cards correctly.

This could also be turned into a competition. The person who draws the largest measurement gets to keep the card. The person with the most cards at the end is the winner.

Decimal fractions are introduced at second level and measurement conversions can be used to complement learning within ‘number and number processes’.

Initially, introduce conversions which use simple decimal fractions (similar to the ones in the image below) and then begin to introduce more challenging conversions.

1 kilogram = 1000 grams	1 kilometre = 1000 metres	1 litre = 1000 millilitres
$\frac{1}{2}$ kg = 500 g = 0·5 kg	$\frac{1}{2}$ km = 500 m = 0·5 m	$\frac{1}{2}$ l = 500 ml = 0·5 l
$\frac{1}{4}$ kg = 250 g = 0·25 kg	$\frac{1}{4}$ km = 250 m = 0·25 km	$\frac{1}{4}$ l = 250 ml = 0·25 l
$\frac{3}{4}$ kg = 750 g = 0· 75 kg	$\frac{3}{4}$ km = 750 m = 0·75 km	$\frac{3}{4}$ l = 750 ml = 0· 75 l

6·5 kilometres	4·5 metres	5·5 kilograms	2·5 litres
↓	↓	↓	↓
6 km + 500 m	4 m + 50 cm	5 kg + 500 g	2 l + 500 ml
↓	↓	↓	↓
6500 m	450 cm	5500 g	2500 ml

Learners may use different steps when making these conversions and exploration of these should be discussed. At second level mixed units should be used less as the end conversion should be displayed in the decimal fraction or the unit, for example, 4.5 m or 450 cm.

6·25 kilometres	4·75 metres	5·25 kilograms	2·75 litres
↓	↓	↓	↓
6 km + 250 m	4 m + 75 cm	5 kg + 250 g	2 ℓ + 750 ml
↓	↓	↓	↓
6250 m	475 cm	5250 g	2750 ml

When learners can successfully carry out conversions similar to the ones above, more challenging conversions can be introduced.

3·375 km	4 m 27 cm	7·925 kg	2 ℓ 521 ml
↓	↓	↓	↓
3 km + 375 m	4 m + 0·27 m	7 kg + 925 g	2 ℓ + 0·521 ℓ
↓	↓	↓	↓
3375 m	4·27 m	7925 g	2·521 ℓ

It is important to spend time specifically exploring conversions which involve zero as a place holder.

2·005 km	3 m 7 cm	6·028 kg	7 ℓ 003 ml
↓	↓	↓	↓
2 km + 5 m	3 m + 0·07 m	6 kg + 28 g	7 ℓ + 0·003 ℓ
↓	↓	↓	↓
2005 m	3·07 m	6028 g	7·003 ℓ

Activities similar to ‘Matching Pairs’ and ‘Greater Than’ (pages 38 and 39) could be used to reinforce conversions using decimal fractions. Learners could also be provided with a range of measurement cards and asked to place in order from shortest to longest, holds the least to holds the most or has the smallest mass to greatest mass.

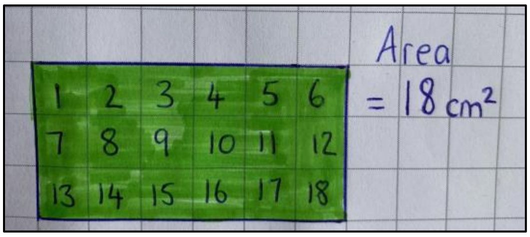
Carrying out measurement conversions requires an understanding of place value and decimal fractions. This may need to be revisited for some learners before they are ready to carry out more complex measurement conversions. Examples of how to support this can be found on pages 34 – 37 of the Number and Number Processes document.

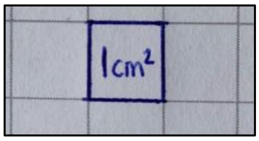
Area

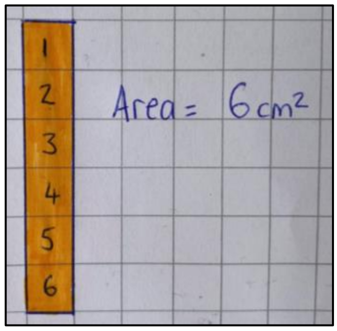
At first level, area was introduced through counting squares and half squares. As learners progress into second level they should be able to explain that area is the flat (2D) space taken up by a shape. They should work towards being able to calculate the areas of a range of squares, rectangles and right-angled triangles. It can be beneficial to revisit counting squares to find the area of shapes using squared paper or Geoboards (see pages 29 – 31).

Square centimetres (cm^2) can be introduced using square centimetre paper**. It is important to note that the correct way of saying this unit of measurement is ‘square centimetres’ and not ‘centimetres squared’.**

Squared Paper

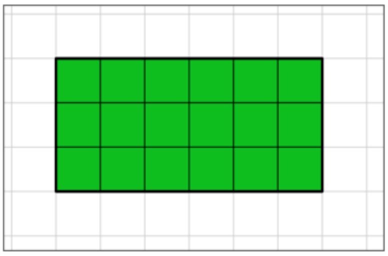






The formula for finding the area of a rectangle can be linked to what learners already know in relation to multiplication and arrays.

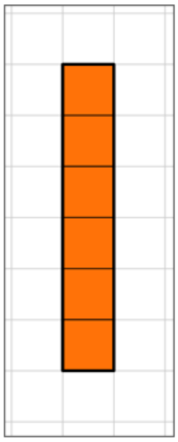
Arrays



Area = 3 rows of 6 squares

= 3 x 6

= 18 cm^2



Area = 6 rows of 1 square

= 6 x 1

= 6 cm^2

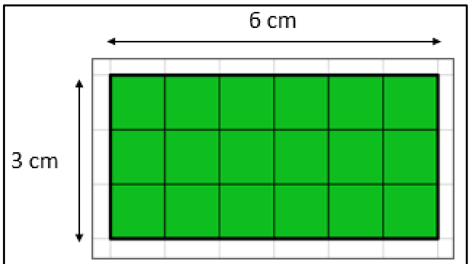
images are not to scale

Area = length x breadth

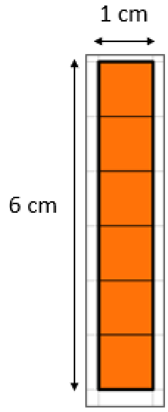
A = l x b

Links can be made with the number of boxes on the centimetre squared paper and the measurements taken using a ruler. This could be done by asking learners to investigate each individual box they are using to measure.

This can then be explored further using larger squares and rectangles and rulers.

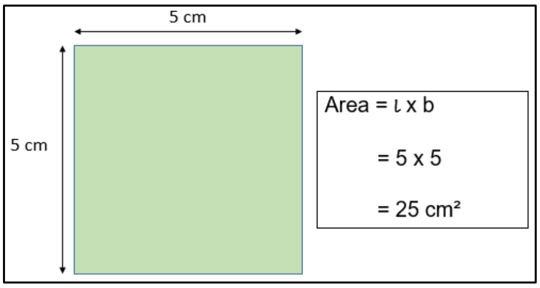


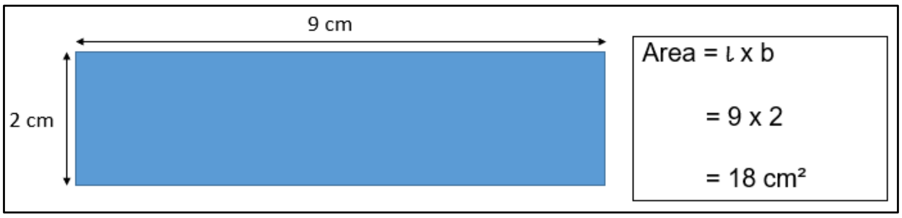
Area = length x breadth
= 6 x 3
= 18 cm^2



Area = length x breadth
= 6 x 1
= 6 cm^2

This can then progress from counting the squares to using a ruler and area formula to calculate the area where the individual squared centimetres are not marked.

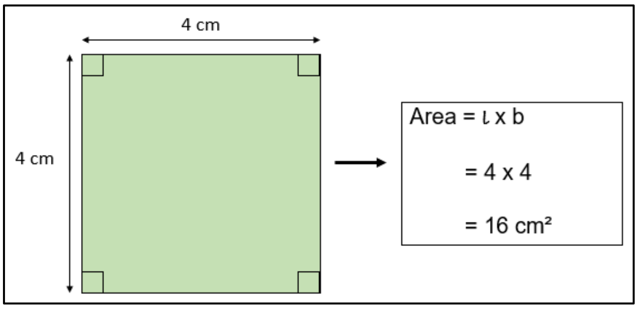




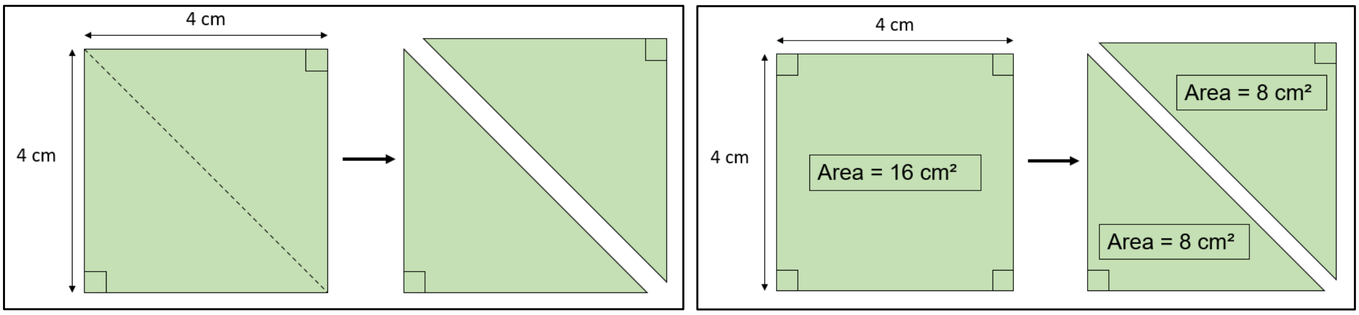
Learners should find the area of familiar objects such as jotters, tablets, tables, etc. Rulers, metre sticks, tape measures and trundle wheels can then be used to measure and calculate the area of items using square millimetres, square centimetres and square metres. Learners can then begin to move onto more abstract calculations of larger squares and rectangles using the area formula.

When learners are able to calculate the area of squares and rectangles they can progress to finding the area of right angled triangles.

Introducing the area of right angled triangles should be closely linked to what learners already know in relation to finding the area of squares and rectangles.

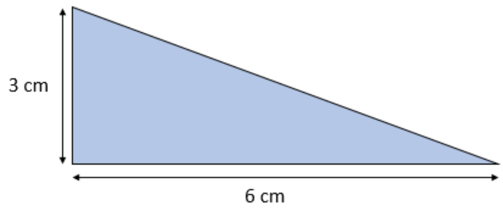


If the area of the 'whole' square is 16 cm^2 then each half must have an area of 8 cm^2.



This can then be repeated with a range of different sized squares and rectangles. Learners can then begin to make the link between this and the formula to calculate the area of a right angled triangle.

$$\text{Area of a right angled triangle} = \frac{\text{length x breadth}}{2}$$



$$\text{Area} = \frac{\text{length x breadth}}{2}$$

$$= \frac{6 \times 3}{2}$$

$$= \frac{18}{2}$$

$$= 9 \text{ cm}^2$$

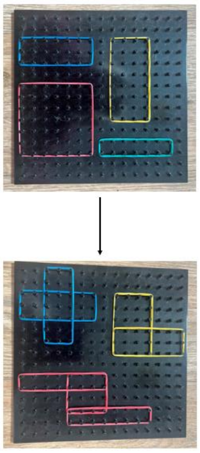
Learners should be provided with opportunities to find the area of a wide range of squares, rectangles and right angled triangles using mm^2, cm^2 and m^2.

Perimeter

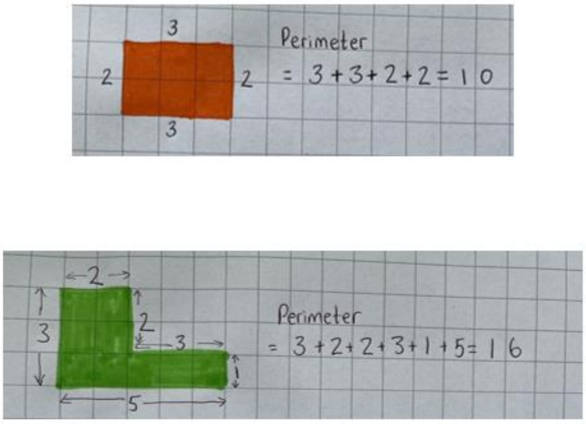
Perimeter is the distance around the edges of a shape and can be found by adding together the length of all the sides of a shape.

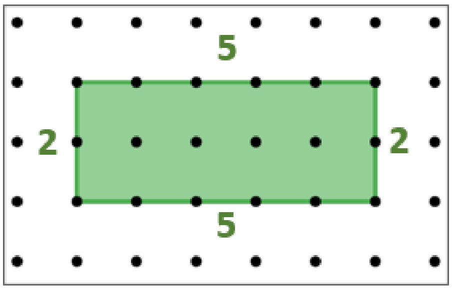
Initially, learners could create shapes using squared or dotty paper and geoboards. They could count the spaces between the pegs on the shapes on the geoboards or count the spaces between lines on the squared paper. When using squared paper, ensure learners understand that they are counting the centimetre-long lines that make up the edges of the shape and not the boxes. Learners could reinforce this understanding by counting steps round part of a playground or using a trundle wheel to measure the perimeter of a sports pitch.

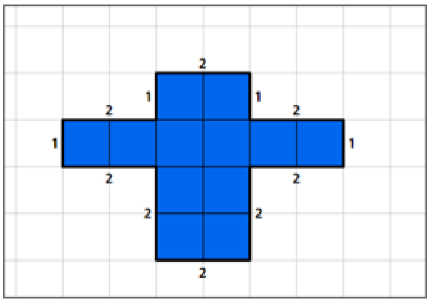
Geoboard



Squared Paper







Digital Tools

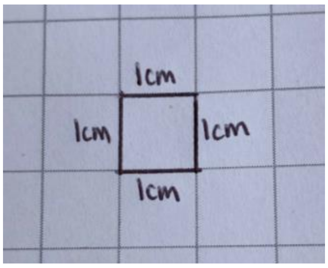
Online tools such as geoboards and perimeter explorers can be used to initially explore perimeter. (Those displayed here are from https://toytheater.com/)

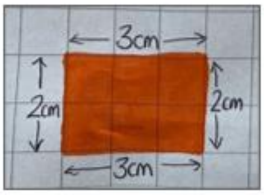
When using Geoboards, ensure that learners are counting the spaces between the pegs/dots and not the number of pegs/dots.

Some digital tablets have measurement tools built in. Whilst these can be useful for supporting estimates it is worth noting that most of these applications are only about 95% accurate.

This should then progress to using formal units of measurement such as millimetres, centimetres, metres and kilometres. It can often be easier to begin with centimetres as centimetre squared paper can support the progression into pictorial representations.

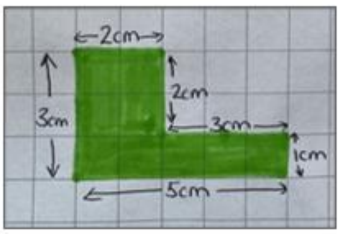
Squared Paper





Perimeter = 3 + 3 + 2 + 2

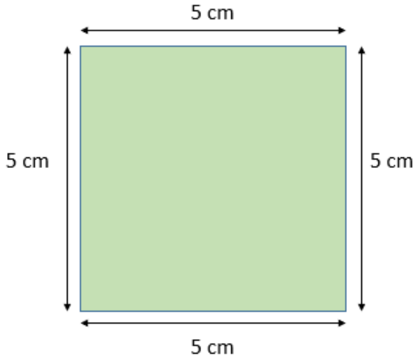
= 10 cm



Perimeter = 3 + 2 + 2 + 3 + 1 + 5

= 16 cm

Once learners are able to find the perimeter of shapes by counting and finding a total length, they can move onto calculating the perimeter of shapes using measurements they have made or carrying out calculations when provided with measurements.



Perimeter = total distance around the outside

= 5 + 5 + 5 + 5

= 20 cm

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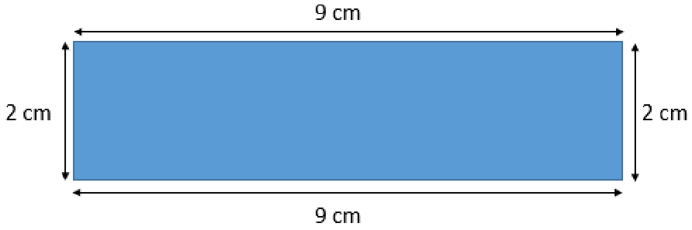


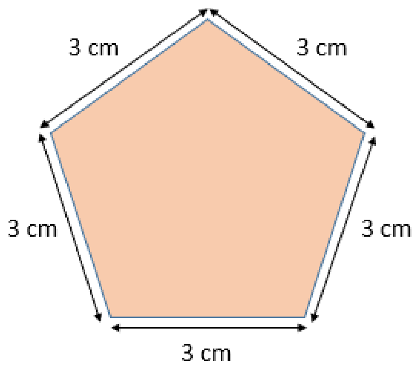
Perimeter = total distance around the outside

= 9 + 9 + 2 + 2

= 22 cm

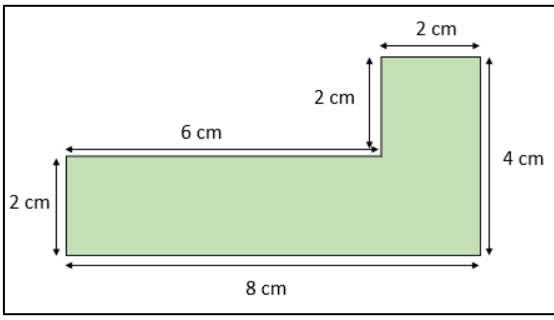
Discussion could take place about any patterns that learners might have started to notice, for example, each side of a square is the same length, therefore the perimeter of a square can be found by multiplying one side by four.

Learners can then move onto shapes with more than four sides, these should be kept to simple straight sided shapes as more complex shapes will be explored in third level.



Perimeter = 3 + 3 + 3 + 3 + 3

= 15 cm



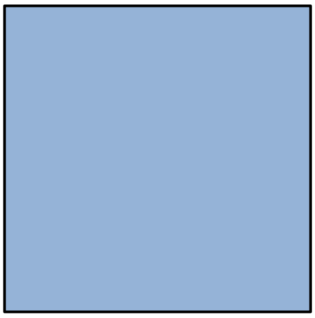
Perimeter = 2 + 6 + 2 + 2 + 4 + 8

= 24 cm

The examples above are using only centimetre measurements, it is important that millimetres, metres and kilometres are also used and specific attention paid to the chosen unit of measurement.

Further challenge could then be given by providing learners with the perimeter and asking them to calculate what the lengths of each side are.

If we know that the perimeter of this square is 28 cm, what is the length of each side?



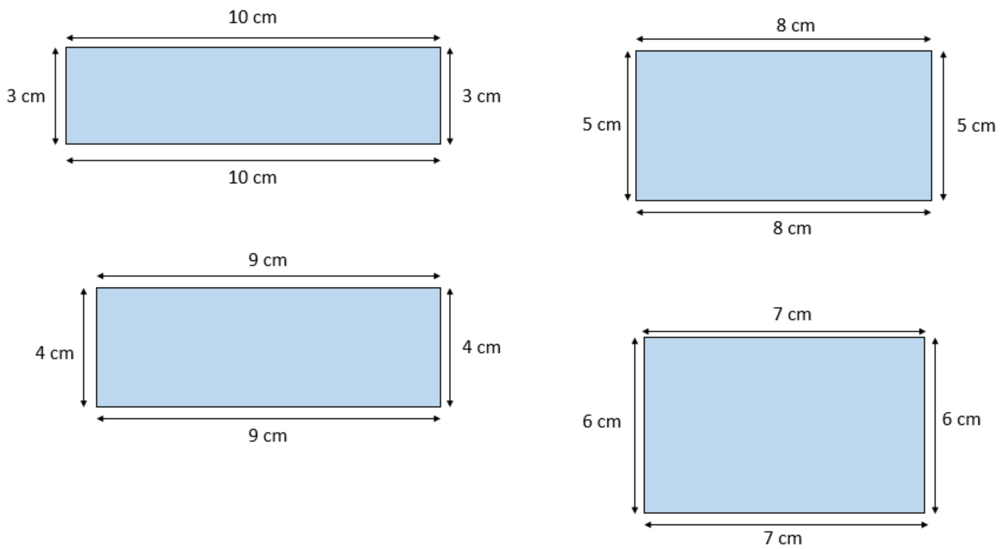
We know that each side is the same length because it is a square. Therefore we can divide the total length by four.

28 cm ÷ 4 = 7 cm

Each side measures 7 cm.

If we know that the perimeter of a rectangle is 26 cm, what could the length of each side be?

This is a bit more challenging as there is more than one possible solution.



Learners should be encouraged to investigate and explore different possible solutions to problems similar to this. They could use pencil, paper and rulers, concrete materials or digital tools such as the area and perimeter explorer on Toy Theatre.

It is important that learners are provided with opportunities to measure the area and perimeter of a wide range of everyday items. Some suggestions are below but there are many more items and spaces which could be measured.

floor of the classroom	windows	doors	
corridors	outdoor play spaces	jotters	tables
sheets of paper	computer screens/boards	flower beds	

Discussion around why we need to be able to measure perimeter is important, for example, to find the lengths of fences, borders, decorative edges, skirting boards etc.

Area and perimeter could then be used together to create different challenges.

Who is correct?

Read the statements from Alexi and Aisha, who is correct? Explain your thinking.

Alexi

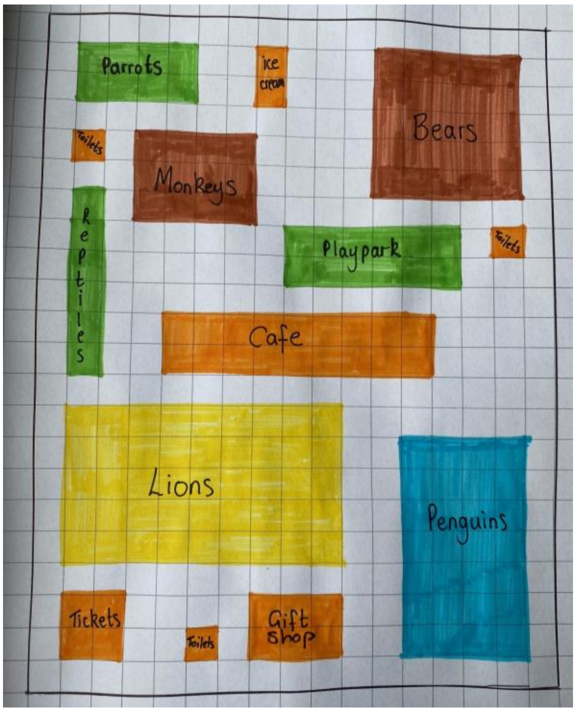
If two rectangles have the same perimeter, they must have the same area.

Aisha

If two rectangles have the same perimeter, they could have a different area.

Plan it out

Learners could be asked to create a plan for a specific area such as a classroom, a zoo, a park, a fairground or anything else of interest. They could use area and perimeter to help them to plan using given criteria for measurements for each item which can be varied depending on where they are in their learning. The illustration below provides one example of what this could look like.



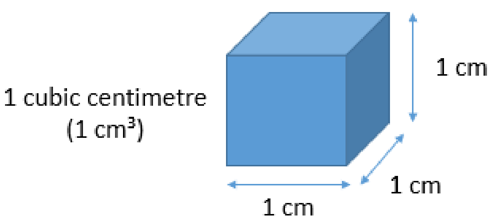
Further differentiation can be provided by:

- providing templates
- using paper to cut out and place items physically
- providing less/more criteria
- using different measurements
- using straight sided shapes other than squares and rectangles

Volume

Volume is the amount of space a 3 dimensional (3D) object takes up. Initially learners could use cubic centimetres to create simple cubes and cuboids and count each cube to find the volume. **It is important to note that the correct way of saying this unit of measurement is 'cubic centimetres' and not 'centimetres cubed'.**

not to scale



4 cubic centimetres
(4 cm^3)

12 cubic centimetres
(12 cm^3)

36 cubic centimetres
(36 cm^3)

Conservation of Volume

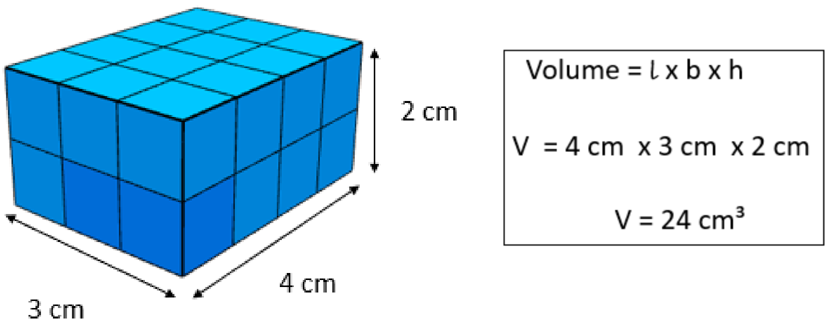
Provide learners with a collection of cubic centimetres and ask them to investigate how many different cubes or cuboids they can make.

Ensure that the number of cubic centimetres provided will allow each learner to create a few different cubes or cuboids.

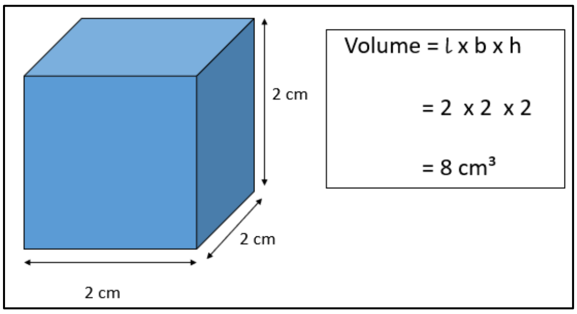
These images represent some of the ways that 24 cm^3 can be arranged.

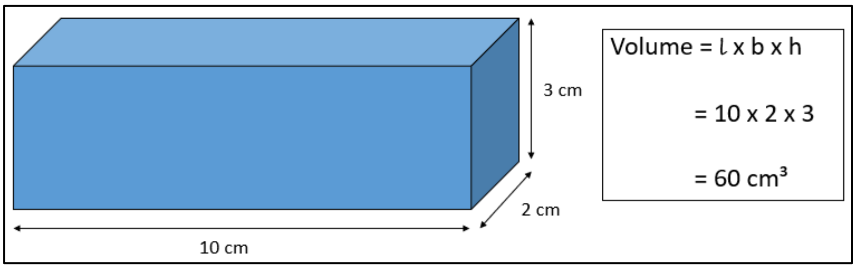
This could be further extended by providing each learner with a different number of cubes and asking them to explore who will be able to make the most cubes or cuboids with them.

When learners can confidently count cubic centimetres to find the volume, the formal calculation can be linked with their previous learning. This is calculated by multiplying **length x breadth x height**. It can be beneficial to continue to use concrete material initially to support learners to make the link between this calculation and the cubes and cuboids that they have been used to working with. To begin with, the number of cubes within each layer could be counted and then the total for each layer added together. Learners should hopefully begin to make connections to previous learning in relation to finding the area of a square or rectangle.

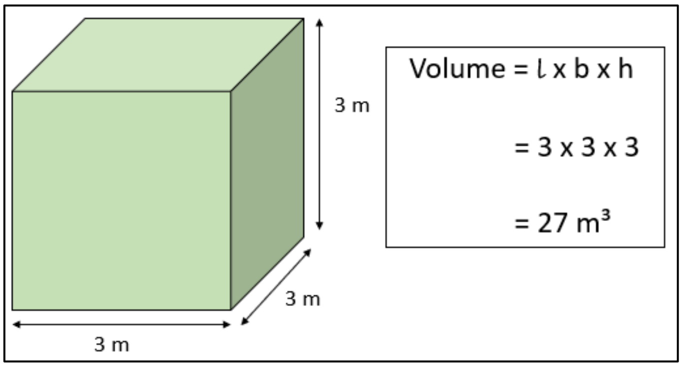


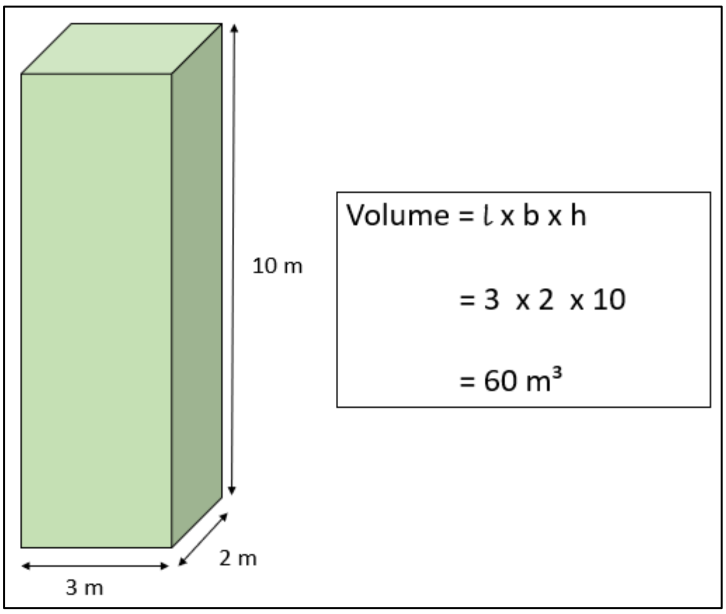
This can then be extended to provide learners with opportunities to calculate the volume of pictorial representations of cubes and cuboids.





This learning can then be extended to include cubic metres (m^3).





Imperial Units

The metric system is the recognised measurement system in Scotland, however, learners are likely to come across imperial measurements at different points in their lives. Some examples of these are noted below but are not limited to these.

Road signs measured in miles.

Cartons of milk are commonly spoken about in terms of 'pints' of milk.

Currently, UK shops are legally required to sell items based on the metric system. However, learners may have been exposed to other imperial measurements.

Examples of these could include:

- 'fluid ounces' in baking and cooking or juice containers
- 'cups', 'tablespoons' and 'teaspoons'
- 'pounds' of sugar, apples,

The speed of travelling vehicles is commonly measured in 'miles per hour'.

The mass of a new born baby is officially recorded in kilograms (kg), however it is often shared in pounds (lbs) and ounces (oz) by family and friends. Additionally, many people record their mass in stones and pounds.

Birth Announcement

Sam and Greg Simm would like to announce the birth of their baby girl

Frances Simm

Born on 31/7/23
2:36 am
7 lbs 5 oz

Measuring the height of people is often referred to in 'feet' and 'inches'.

Learners would **not** be expected to use the imperial system to measure items or to make conversions between the imperial and metric system, they would only be expected to develop an awareness of it. Dialogue could focus on learners' own experiences around this and the use of imperial units within the local and wider context could be explored.

Points to consider:

- Carrying out measurement conversions requires an understanding of place value and decimal fractions. This may need to be revisited for some learners before they are ready to carry out more complex measurement conversions.

- Learners are not expected to use the imperial systems to measure items or make conversions between the imperial and metric systems, they are only expected to develop an awareness of the imperial measuring system.
- Learners should be encouraged to make estimations before carrying out any type of measuring activity.

Reflective questions:

- **How can we encourage learners to make estimates before they measure items and then compare with the actual measurements?**
- **How often do we provide opportunities for learners to select the most appropriate tool for measurement and use a variety of different tools?**
- **In what ways can we support learners to develop an understanding of conservation of measurement?**

Links across the curriculum:

Numeracy and Mathematics

- Number
  1. Whole numbers
  2. Place value, including zero as a place holder and decimal fractions
  3. Addition, subtraction, multiplication and division

- Estimation – estimation in the context of measure

- Fractions, decimal fractions and percentages
  1. equivalent forms of common fractions, decimal fractions and percentages

- Properties of 2D shapes and 3D objects
  1. measuring length, width, area and perimeter of 2D shapes
  2. calculates the volume of cubes and cuboids

Technologies (Food and Textile Technology)

- Practical skills and cooking techniques, e.g. weighing and measuring
- Designing and constructing models/product
- Representing ideas, concepts and products through a variety of graphic media

Science

- Inquiry & investigative skills
  1. makes predictions about the scientific investigation
  2. observes and collects information and makes measurements using appropriate devices and units

Third Level

At third level and beyond it makes sense to consider both the numeracy **and** mathematics 'Measurement' experiences and outcomes when planning for learning, teaching and assessment therefore both are included in this document. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

The table below includes the **numeracy** experiences and outcomes related to 'Measurement' at third level.

Experiences and Outcomes	Benchmarks
I can solve practical problems by applying my knowledge of measure, choosing the appropriate units and degree of accuracy for the task, and using a formula to calculate area or volume when required. **MNU 3-11a**	• Chooses appropriate units for length, area and volume when solving practical problems. • Converts between standard units to three decimal places and applies this when solving calculations of length, capacity, volume, and area

The table below includes the **mathematics** experiences and outcomes related to 'Measurement' at third level.

Experiences and Outcomes	Benchmarks
Having investigated different routes to a solution, I can find the area of compound 2D shapes and the volume of compound 3D objects, applying my knowledge to solve practical problems. **MTH 3-11b**	• Calculates the area of a 2D shape where the units are inconsistent. • Finds the area of compound 2D shapes constructed from squares, rectangles, and triangles. • Finds the volume of compound 3D objects constructed from cubes and cuboids.

Effective learning and teaching approaches

Problems set in the context of measurement provide rich opportunities to develop skills across many different numeracy and mathematics curricular organisers. These include, but are not restricted to, estimation and rounding, number and number processes, fractions, decimal fractions and percentages and data analysis.

Units of Measurement

Learners should now be familiar with a wide variety of units of metric measure for length, area and volume and should be able to choose the most appropriate unit and degree of accuracy necessary to solve a variety of practical problems. Although mass is not mentioned explicitly it can provide a useful context for learning.

Mixed up Measure.

Learners could be given a set of cards like those shown below and asked to rearrange them to find a combination where every measurement is valid.

Area of a football pitch	2	km
Mass of a bus	4	tonnes
Area of a postage stamp	77910	g
Area of Scotland	7140	km^2
Mass of a crisp	24	cm
Mass of a baby	1120	m^2
Length of a marathon	12	kg
Height of a pet cat	42	mm^2

Expressing measurements in related units.

At third level learners should continue to develop fluency and accuracy when expressing measurements in related units. This should continue to be linked to practical examples as frequently as possible and learners should be provided with opportunities to convert both ways, for example ml to L and L to ml. Activities such as “Matching Pairs” and “More

Than" which are described on pages 37 and 38 in the second level section of this document can be adapted to provide suitable challenge.

Discussion of related metric units and the standard metric prefixes kilo, centi and milli, with which learners are already familiar, will enable learners to develop a deeper understanding of the relationships between related units.

Quantity	Unit	Symbol
length	metre	*m*
mass	gram	*g*
capacity	litre	*ℓ*

Prefix	Symbol	Factor	
kilo	k	1000	thousand
centi	c	0.01	hundredth
milli	m	0.001	thousandth

Learners will already be aware that for smaller or larger quantities, we can adapt the base units of measurement shown in the first table using the prefixes shown in the second table to create, for example kg, cm, and ml.

The range of unit prefixes in common use goes well beyond the examples of k, c, and m. Very large or very small units of measurement which use prefixes such as those shown in the table below could be explored.

Prefix	**Unit Abbrev.**	**Meaning**
giga	G	1,000,000,000
mega	M	1,000,000
kilo	k	1000
hecto	h	100
deka	da	10
		1
deci	d	1/10
centi	c	1/100
milli	m	1/1000
micro	µ	1/1,000,000
nano	n	1/1,000,000,000
pico	p	1/1,000,000,000,000

Learners could be asked to think of examples of units of measurement that they have used or seen in use in other curricular areas or everyday situations. For example, learners may have used gigawatts, megavolts, or nanometres in Science. Although Scientific Notation is not formally introduced until fourth level it might be useful at this point to begin to look at how the numbers in the third column can be expressed using powers of 10. Expressing large and small numbers in this way can be helpful in developing an understanding of scale. Learners may also have come across this format in science and technologies or even on their scientific calculator display.

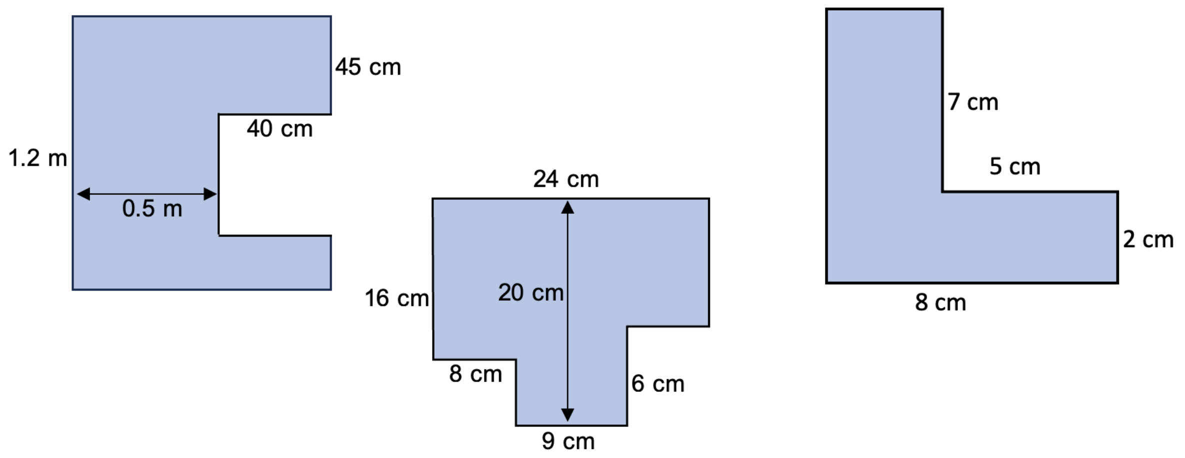
Learners are also likely to have come across prefixes used in the context of digital technology (e.g. gigabyte). It should be made clear that these are not decimal prefixes, rather these mean the power of 2 that is closest to the decimal value.

At second level learners will have had the opportunity to measure the perimeter of simple 2D shapes, will have found the area of squares, rectangles, and right-angled triangles by initially counting squares and will have estimated volume by counting cubes before moving on to using appropriate formulae. It is important that learners have a secure understanding of these concepts and are not merely memorising the formulae and applying a rule. This conceptual understanding will allow learners to develop these skills at third level to tackle practical problems involving more complex shapes and objects.

Perimeter

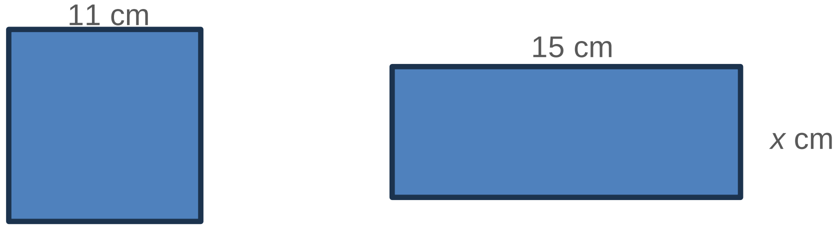
Learners should be provided with opportunities to solve a range of problems involving the perimeter of rectangular compound shapes. Learners should be given examples where lengths need to be found using mathematical reasoning and this should also include problems where units are inconsistent.

Find the perimeter of each of these shapes.



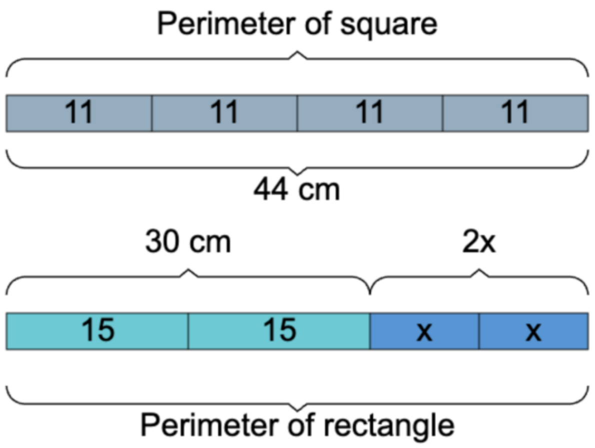
Further challenge could be provided by using perimeter problems as a context to develop algebraic reasoning.

Learners could be asked to consider the following shapes:



The perimeters of the square and the rectangle are the same.
Find the width, x, of the rectangle.

The problem could then be modeled pictorially using, for example, a bar model:



This can also be recorded symbolically as:

$$44 = 30 + 2x$$
$$14 = 2x$$
$$7 = x$$

<u>Solution: The width of the rectangle is 7 cm</u>

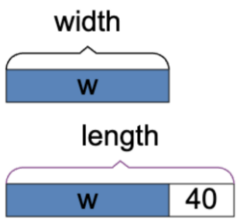
Consider the tabletop below:

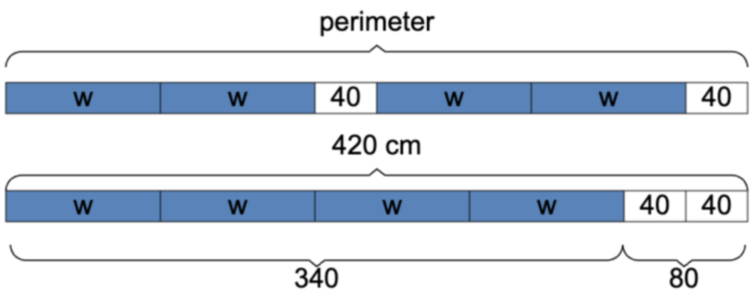
The length of the tabletop is 40 cm more than the width.

Its perimeter is 4.2 metres.

Find the length and the width of the table.

Let the width = w Then the length(l) = w + 40





4.2 m = 420 cm

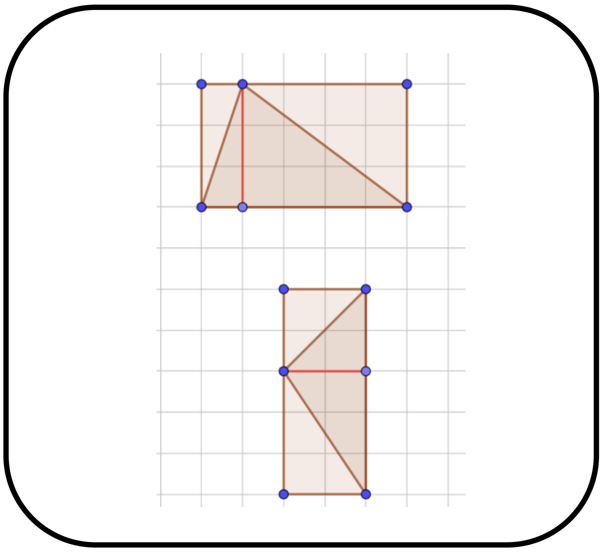
$$4w + 80 = 420$$
$$4w = 340$$
$$w = 85$$

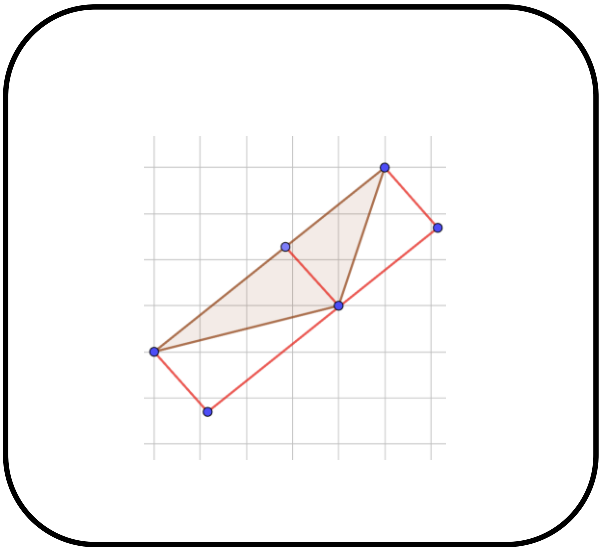
l = 85 + 40 = 125 cm

<u>Solution: The rectangle has width = 85 cm and length = 125 cm</u>

Area

As learners progress on to third level they should be able to apply their knowledge of area of squares, rectangles, and right-angled triangles to calculate the area of a wide range of compound 2D shapes.
Learners can investigate the area of non-right-angled triangles by drawing, enclosing in a rectangle and partitioning into right-angled triangles.

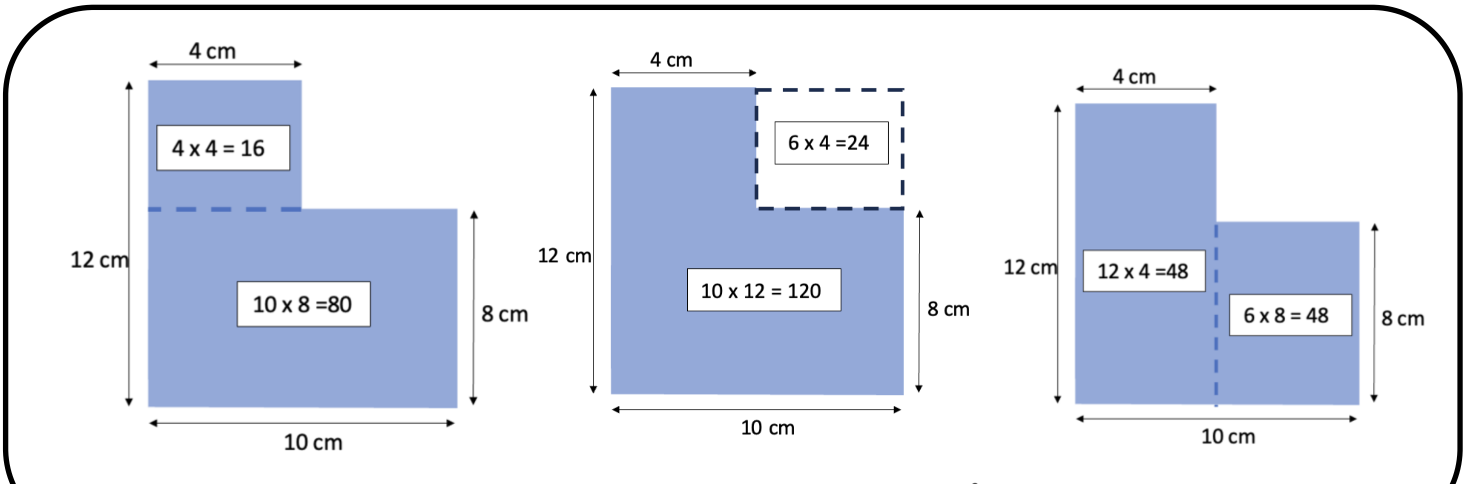




It is important that learners understand that any triangle can be placed inside a rectangle and, by partitioning, as shown in the examples above, they should recognise that each part of the triangle is half of the smaller rectangle in which it sits. Therefore, the area of the whole triangle is half that of the large rectangle. It is also important that learners understand that the base and height of the triangle are perpendicular. This will then lead them to the generalisation that for any triangle:

$$\text{Area} = \frac{1}{2}\,\text{base} \times \text{height}$$

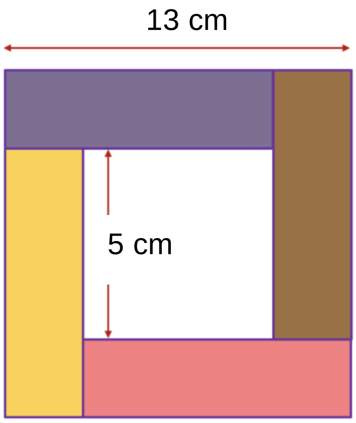
Learners should be given the opportunity to explore area problems where there is more than one route to a solution including those exploiting missing areas. Learners could be given a small number of examples and be asked to identify at least two ways that the area could be calculated.



Area = 80 + 16 = 120 – 24 = 48 + 48 = 96 cm^2

Learners should be given the opportunity to engage in a wide range of different types of tasks in the context of area and perimeter which will allow them to apply their knowledge and develop their mathematical reasoning skills.

Learners could be asked to consider the following:



What is the area of each congruent rectangle?

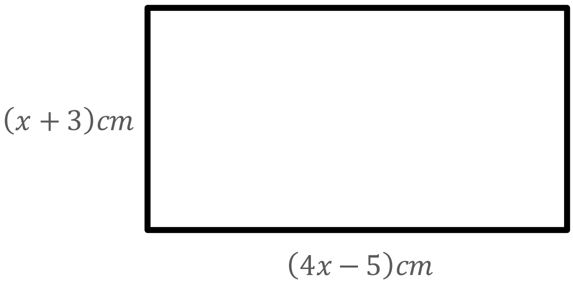
Can you find a rectangle with a side length of 10 cm whose perimeter and area have the same numerical value?

Can you find more examples of rectangles where this is true?

I'm thinking of a rectangle with an area of 24cm^2. What could its perimeter be?

Area and perimeter can again provide a useful context to support algebraic reasoning. Learners should be given the opportunity to explore problems such as:

This rectangle has length $(4x - 5)$ cm and width $(x + 3)$ cm



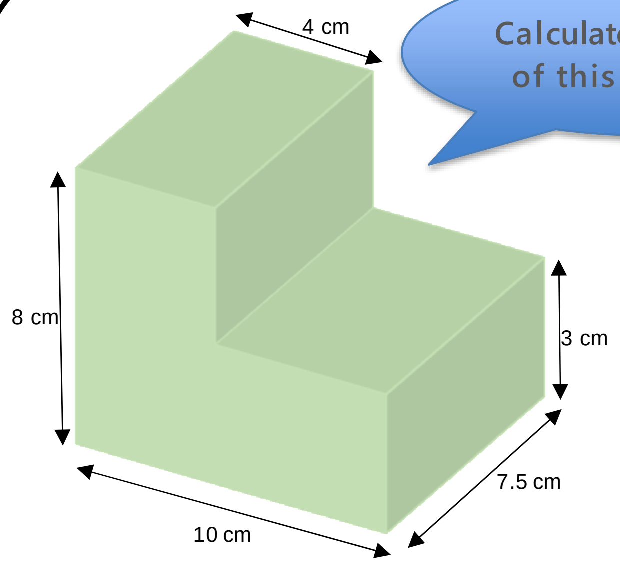
Find the value of for which the perimeter in 26 cm.

Volume

Learners will be familiar with finding the volume of cubes and cuboids from second level and will have been introduced to the formula:

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}$$

At third level learners should be presented with a variety of problems which require them to calculate the volume of compound 3D objects constructed from cubes and cuboids. Learners should be given the opportunity to explore problems where there is more than one route to a solution, including those exploiting missing volumes. Learners could be given a small number of examples and be asked to identify at least two ways that the volume could be calculated.



Calculate the volume of this 3D object.

Volume = (10 × 3 × 7.5) +(4 × 5 × 7.5)
= 375 cm^3

Can you find another way to calculate it?

Volume = (4 × 8 × 7.5) + (6 × 3 × 7.5)
= 375 cm^3

Is there another way?

Volume = (10 × 8 × 7.5) – (6 × 5 × 7.5)
= 375 cm^3

Calculations could be set in a variety of contexts. Some examples are given below:

I have a parcel which had a volume of 24 cubic cm. What could its dimensions be?

A cuboid has sides of length x cm, x cm and $3x$ cm.
Its volume is 1029 cm^3.
Find the value of x.

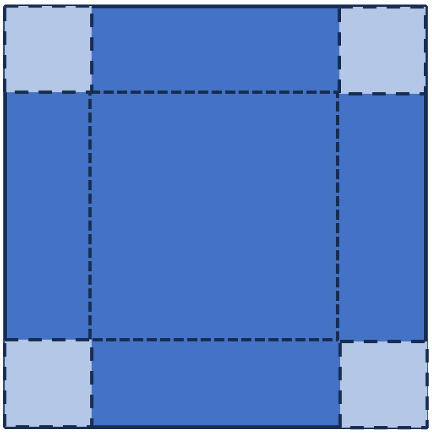
The dimensions of an Olympic-sized swimming pool are 50 m long, 25 m wide and the water is 2 m deep.

The swimming pool is to be completely drained at a rate of 10 litres per second.

How long will it take to fully drain the pool?

Maximum Volume Challenge

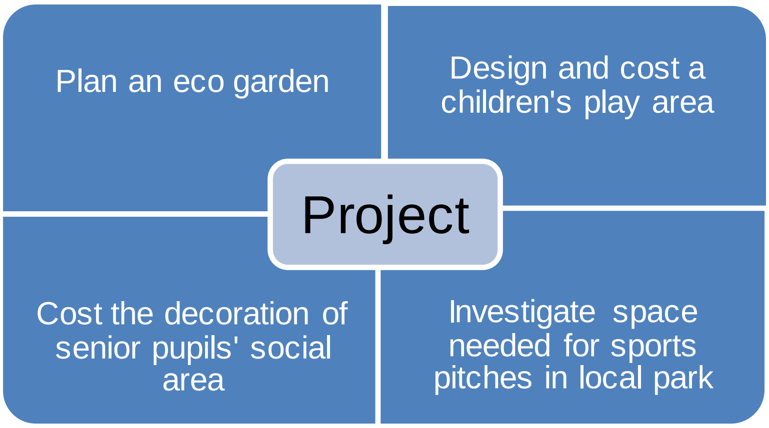
Learners could be given a 20 cm by 20 cm square piece of card and asked to cut out four equal squares from the corners and fold up the flaps to make an open box (no lid).



They could then investigate the volume of the box they have created. Prompt questions could include:
What is the volume of your box?
If you cut out a different size of squares what would your volume be?
What is the largest volume you can make?

Projects and practical activities

A wide variety of practical hands-on activities and projects could be undertaken which allow learners to apply their knowledge of measurement.



Projects can provide learners with opportunities to apply skills from a variety of other aspects of the numeracy and mathematics curriculum such as scale drawing, budgeting as well as developing meta skills such as communication and collaboration.

Points to consider:

- Calculation of perimeter, area and volume should not just be about memorising formula and applying set procedures. Learners should be presented with problems which allow them to develop a deep understanding of the concepts involved.
- Understanding of measurement and an ability to carry out calculations accurately is an important skill for future life, learning and work and can provide a familiar context to explore other areas of numeracy and mathematics.

Reflective questions:

- **How can we ensure that learners at third level continue to have the opportunity to carry out practical tasks related to measurement?**
- **How can we use the context of measurement to develop understanding in other aspects of numeracy and mathematics?**
- **How can we use concrete materials and pictorial approaches to model problem solving strategies?**

Links across the curriculum:

Numeracy and Mathematics

- Estimation and rounding
  - estimation in the context of measure
  - rounding to three decimal places
- Number - Addition, subtraction, multiplication, and division
- Fractions, decimal fractions, and percentages - working with decimal fractions to 3 decimal places.
- Money
  - best value
  - budgeting
- Angle, symmetry, and transformation - scale drawing
- Time - Using measurements to derive related quantities, e.g., speed.

Technologies

- Food and Textile Technology - Practical skills and cooking techniques, e.g., weighing and measuring.
- Craft, Design, Engineering and Graphics - designing and constructing models/products.

Science

- makes accurate measurements using appropriate devices and units.

Fourth Level

At fourth level in addition to the **numeracy** and **mathematics** ‘Measurement’ experiences and outcomes we will consider those relating to calculating the circumference and area of a circle. The experiences and outcomes should be used in the planning of learning, teaching and assessment. It is important to note that the benchmarks are designed to support teacher professional judgement in progress towards and achievement of a level. There are a range of different experiences that learners need to be exposed to before these can be achieved.

The table below includes the **numeracy** experiences and outcomes related to ‘Measurement’ at fourth level.

Experiences and Outcomes	Benchmarks
I can apply my knowledge and understanding of measure to everyday problems and tasks and appreciate the practical importance of accuracy when making calculations. **MNU 4-11a**	• Demonstrates understanding of the impact of truncation and premature rounding.

The table below includes the **mathematics** experiences and outcomes related to ‘Measurement’ at fourth level.

Experiences and Outcomes	Benchmarks
Through investigating real-life problems involving the surface area of simple 3D shapes, I can explore ways to make the most efficient use of materials and carry out the necessary calculations to solve related problems. **MTH 4-11b** I have explored with others the practicalities of the use of 3D objects in everyday life and can solve problems involving the volume of a prism, using a formula to make related calculations when required. **MTH 4-11c**	• Calculates the area of kites, parallelograms, and trapeziums. • Uses formulae and calculates the surface area of cylinders, cuboids, and triangular prisms. • Calculates the volume of triangular prisms and cylinders using formulae.

The table below includes the Properties of 2D shapes and 3D objects **mathematics** experiences and outcomes related to the circle at fourth level.

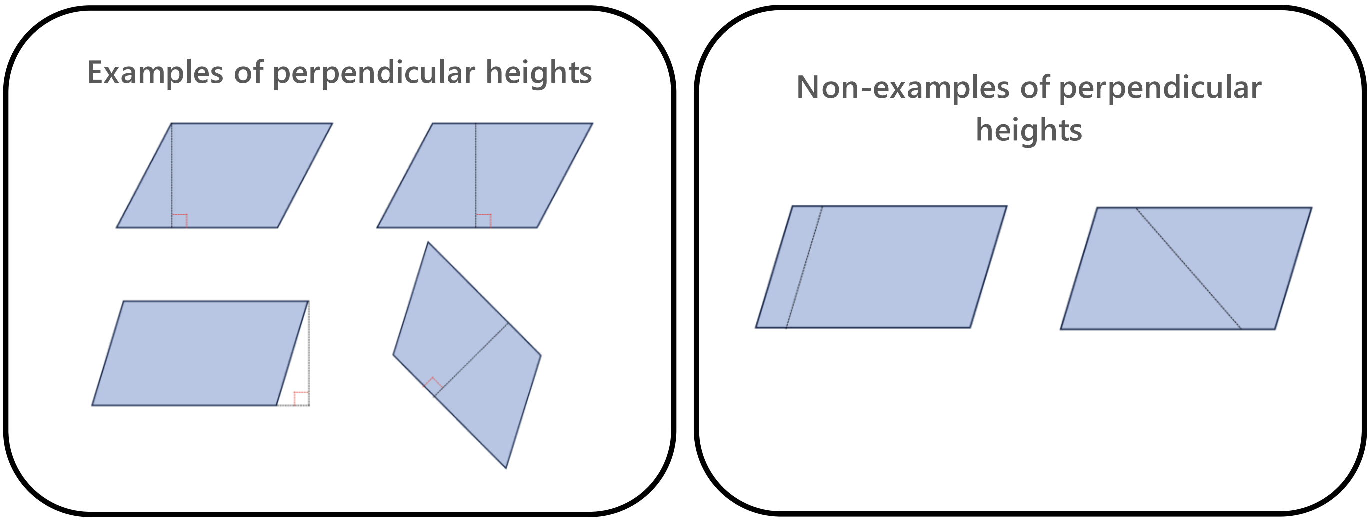
Experiences and Outcomes	Benchmarks
Having investigated the relationships between the radius, diameter, circumference, and area of a circle, I can apply my knowledge to solve related problems. **MTH 4-16b**	• Uses the formula $C = \pi D \text{ or } C = 2\pi r$ to calculate the circumference of a circle. • Uses the formula $A = \pi r^2$ to calculate the area of a circle. • Calculates diameter and radius of a circle when given the area or circumference.

Effective learning and teaching approaches

When learners are confidently working with the 2D shapes and 3D objects discussed at third level then they can begin to use that knowledge in more complex problems and to develop their understanding of how to calculate the area of a range of quadrilaterals, the volume of prisms and cylinders and the surface area of these and cubes and cuboids. It is important to emphasise that formulae developed at this level should not be viewed as unconnected pieces of knowledge but rather a series of consequences of what learners already know. Although the perimeter and area of a circle are not included in the Measurement experiences and outcomes it makes sense to consider these in this document too. It will then be possible to further develop skills in calculating the perimeter and area of compound 2D shapes to include circular properties.

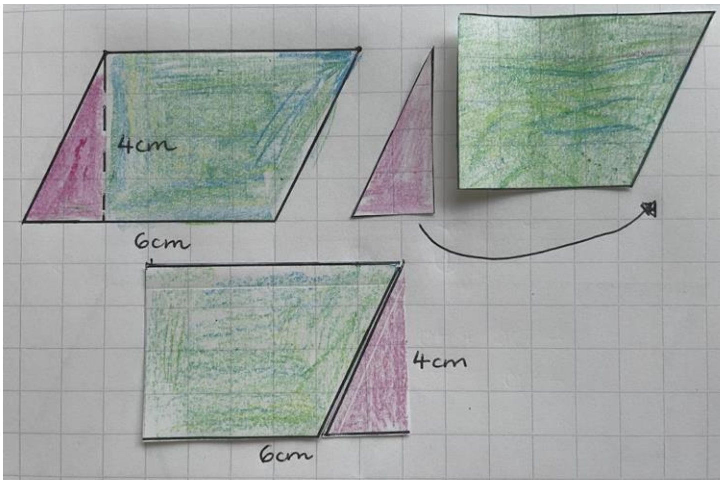
Area of parallelograms and kites

Before moving on to investigate the area of parallelograms learners need to be familiar with their properties and should be able to identify the perpendicular height. Examples and non-examples could be used to consolidate understanding of this.

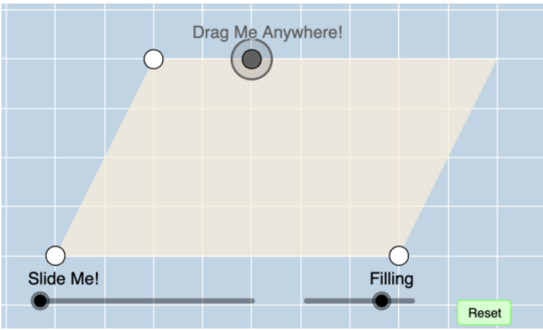


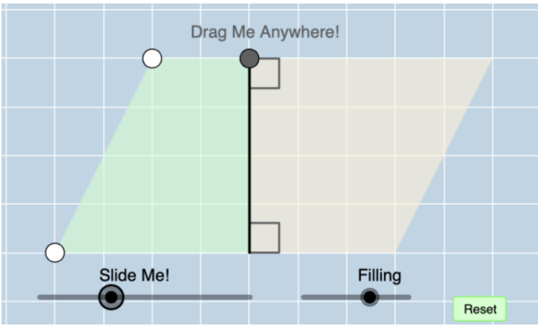
Learners can then investigate the area of a parallelogram using either squared paper and scissors or one of the online tools available, leading to the generalisation that a parallelogram has the same area as a rectangle which has the same base and perpendicular height.

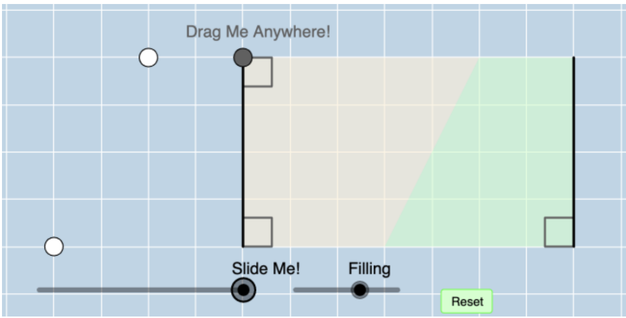
Squared Paper



Digital Tools



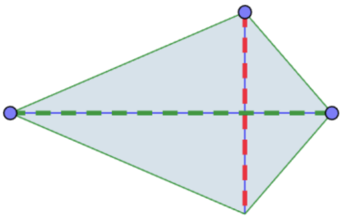


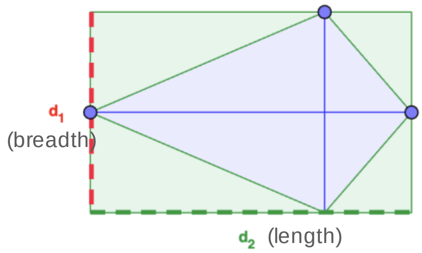


These images are taken from GeoGebra https://www.geogebra.org/m/D8rjsGzF

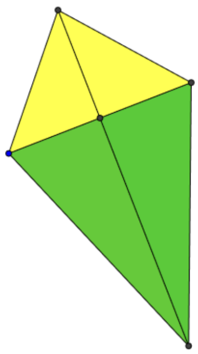
Learners can use their knowledge of area of a triangle and an investigative approach to develop formulae for the area of kites and rhombuses.

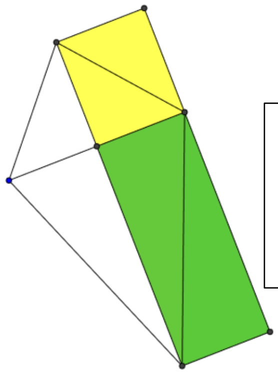
By enclosing in surrounding rectangle





Or by rearranging parts into a rectangle which is half the area of the surrounding rectangle





Leading to the formula:

$$Area = \frac{diagonal_1 \times diagonal_2}{2}$$

These images are taken from GeoGebra https://www.geogebra.org/

Area of a trapezium

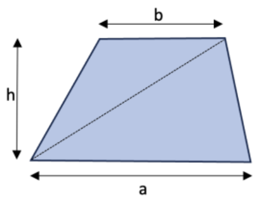
When calculating the area of a trapezium, learners may think that the only method is to divide the trapezium into rectangles and triangles, based on their experiences of finding the area of rectangles and triangles at second and third levels. If they are then presented with the formula, they might simply memorise it, and substitute numbers into it. They may not appreciate the importance of the pair of parallel sides and the significance of the $\frac{1}{2}$ and this can often lead to errors in application.

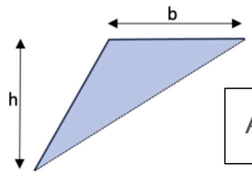
$$\text{Area of a trapezium} = \frac{1}{2}(a + b)h,$$

where a and b are the lengths of the parallel sides and h is the perpendicular height.

By exploring different ways to derive the formula learners can develop a clearer understanding of how to use it correctly.

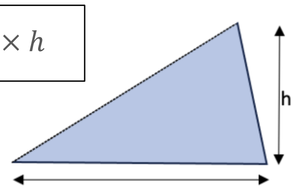
Partition a trapezium into two triangles.





$\text{Area} = \frac{1}{2}b \times h$

$\text{Area} = \frac{1}{2}a \times h$

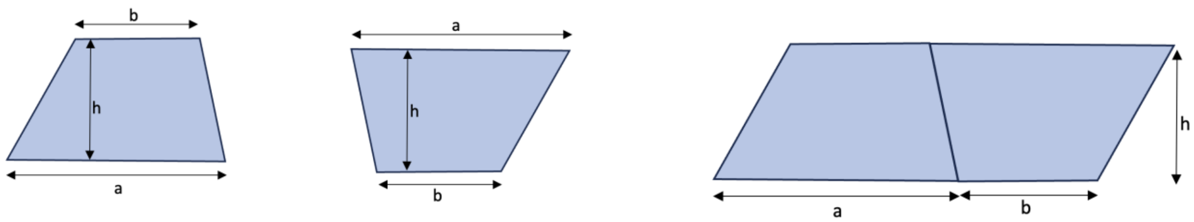


This representation supports learners' understanding of the significance of the 'perpendicular height' part of the formula as they realise that the area of the trapezium is the sum of the area of two triangles.

$$\text{Area} = \frac{1}{2}a \times h + \frac{1}{2}b \times h = \frac{1}{2}(a+b)h$$

Learners could also draw and cut out two identical trapezia and then using their knowledge of angles and tessellation could investigate the representation below.

Use two congruent trapezia to construct a parallelogram.



This representation supports learners' understanding of the significance of the $\frac{1}{2}$ and the 'sum of the parallel sides' parts of the formula. Learners should be able to see that, since the area of the parallelogram = (a + b)h, the area of the trapezium must be half of that.

The circle

Although the relationships between the radius, diameter, circumference and area of a circle are not included in the Experiences and Outcomes until fourth level it is worth considering carefully when these concepts are introduced. If the concept of the area of a circle is introduced at the same time as the circumference of a circle, then some learners may confuse the meanings and formulae. This can be avoided by introducing the circumference of a circle when thinking about perimeter alongside other shapes, and the area of a circle when considering area of kites, parallelograms, and trapezium.

Circumference

Learners should be given the opportunity to explore the relationship between the circumference and diameter of a circle. The key understanding is that no matter how large or small the circle, the ratio of the circumference to the diameter is always the same.

Exploring everyday objects.

Learners could be provided with a variety of circular or cylindrical objects, a piece of string or a flexible measuring tape. They could then work in small groups or pairs to measure and record the length of the diameter and the circumference for each object.

object	circumference	diameter	

Initially the learners could be asked what they notice about their measurements but if a prompt is required then they could be asked to calculate the circumference divided by the diameter for each object.

When learners have identified that the ratio of circumference to the diameter is approximately 3, they can then, by examining the average for each group, begin to identify a more accurate approximation. It is important to discuss outliers and check for errors of measurement or calculation and to explore how choice of units and measuring accuracy will impact on results.

An alternative to measuring objects might be to provide learners with a few different sized circles and ask them to use counters or cubes to measure the diameter and circumference.

There are many excellent online tools and videos which explore and demonstrate the relationship between circumference and diameter and explain the history behind the increasingly efficient and accurate estimation of pi. A good starting place might be the video Pi-Numberphile on YouTube or What is Pi? | Pi Day .

This multiplicative relationship within every circle will then lead to the formulae:

$$Circumference = \pi \times Diameter$$

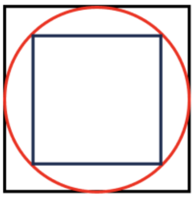
$$or\ \pi = \frac{Circumference}{Diameter}$$

Learners should also understand the multiplicative relationship between any two circles. If one circle has a diameter n times the length of another, then its circumference will also be n times the circumference of the other.

Area of a circle

An investigative approach to deriving the formula for area of a circle will help learners to see why the formula works and give them a visual reminder to the formula if they forget it. Learners could initially be provided with a few circular objects and asked to estimate the area of each. This task could be left open, or prompts could be provided if required. Possible approaches might be:

- Learners can trace the shape of their object on a piece of centimetre grid paper and count how many square centimetres make up the total area of the circle.
- Learners can divide the circle into equal wedges by drawing radii. They can approximate the area of each wedge using the area of a triangle formula.
- Drawing a square or other polygon inside the circle and then the same shape outside the circle to get lower and upper estimates, respectively (a sample drawing of this method like the one below could be shared).

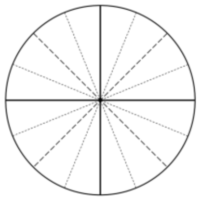


If learners are using the second or third approach, then they could be encouraged to think about their estimates in terms of radius (r). For example, from the circle in square method they could conclude that $2r^2$ < Area of circle < $4r^2$. This might lead to the idea that the average of these, $3r^2$, would be a reasonable approximation for the area of the circle. This could then be tested for the circles estimated using other methods.

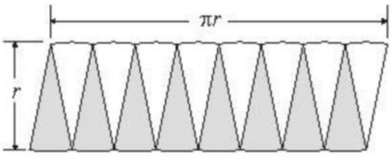
For a connection to mathematical history, you may want to include a brief overview of Archimedes and his method for calculating the area of a circle.

Practical activity to discover the formula area of a circle.

Learners draw and cut out a circle or they could be provided with ready prepared circles or templates.



The circle is then cut into quarters and sixteenths and then begin to arrange so that they alternately point up and down.



Ask learners 'What shape does this resemble?' 'What would happen if we repeatedly cut each wedge in half and rearranged?

You could ask learners to divide the wedges further and see what it looks like.

Thinking about the dimensions and area of their rectangle will lead to the formula for area of a circle:

$$A = \pi r^2$$

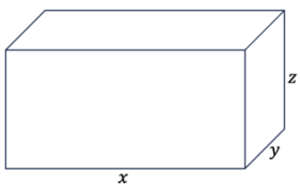
When the formula is established, learners should be presented with problems in a variety of contexts including those which involve calculating the diameter or radius given the area of the circle.

Surface area

Learners may have difficulty visualising how many surfaces cubes, cuboids and triangular prisms have. The opportunity to handle 3D models and to describe how many faces there are and what shape they are is important.

$\mathrm{SA} = 2(xy + xz + yz)$ or
$\mathrm{SA} = 2xy + 2xz + 2yz$

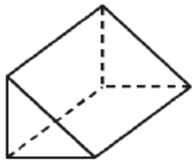
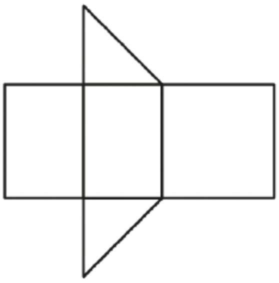
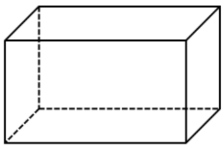
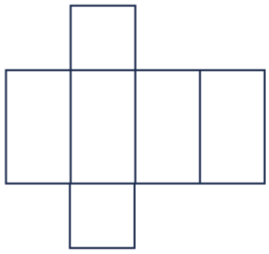
for the surface area of a cuboid, where x, y and z are its dimensions)



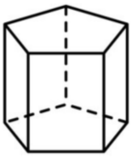
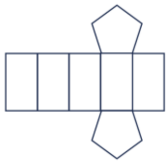
Unthinking memorisation and use of formulae such as the one on the left should be avoided, as this can lead to over-generalising that the surface area of all prisms can be calculated in this way.

It is important for learners to appreciate the structure of all surface area calculations, so that as well as understanding how to work with cubes, cuboids, and triangular prisms at fourth level, they are able to extend their thinking to deal with any prism in the future. They should be encouraged to identify the shapes and dimensions of all the surfaces. For any prism two of these will be the ends and will, therefore, have the same area. There will also be a number of rectangular faces depending on the type of prism.

A simple sketch of each of the faces of the object can help identify shapes and dimensions. This could also be a good opportunity to practise drawing nets in a relevant context.

Prism	Net	Faces
 **Triangular prism**		**Five faces (Two triangular ends and three rectangular faces)**
 **Cuboid (rectangular prism)**		**Six faces (Two rectangular ends and four rectangular faces)**

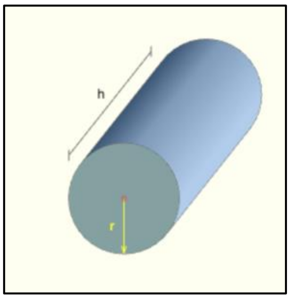
Learners could extend this thinking beyond fourth level to investigate other prisms such as the pentagonal prism below.

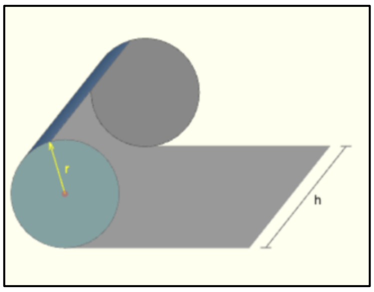
Prism	Net	Faces
		**Seven faces (Two pentagonal faces and five rectangular faces)**

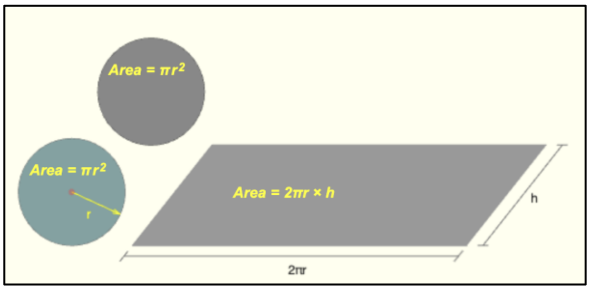
Although cylinders are not strictly speaking prisms, learners should appreciate that the same approach to calculating the surface area can be taken. An understanding of the formulae for area and circumference of the circle can then be applied to the net to develop the formula for the total surface area of the cylinder.

Surface area of a cylinder.

A number of online tools are available which illustrate the derivation of the formula for the total surface area. The images below are from mathopenref.com







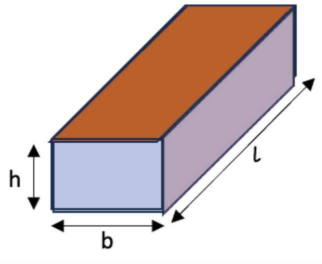
Leading to the formula:

Total surface area	= Area of 2 circular faces + Area of curved surface $= 2\pi r^2 + 2\pi r h$

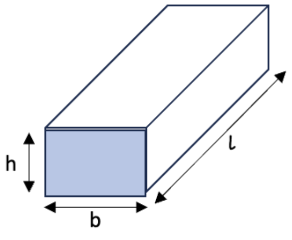
Volume

Learners will be familiar with calculating volume of cubes and cuboids and will have used the formula $\text{Volume} = \text{length x breadth x height}$ at third level. Before moving on to look at the volume of triangular prisms and cylinders it might be useful to revisit this, encouraging learners to think about this as initially calculating the area of one of the faces and then multiplying by the remaining dimension to find the volume.

For example, thinking about this cuboid:

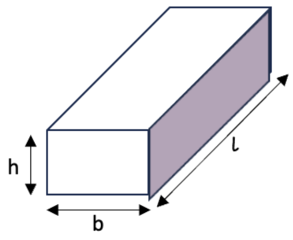


There are various ways of calculating its volume:



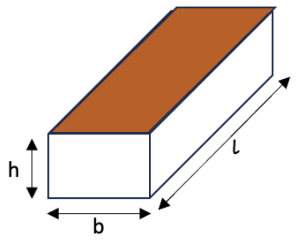
We could find the area of the blue face (b x h) and multiply by the length(l).

Alternatively, we could:



- Find the area of the purple face (l x h) and multiply by the breadth(b).

Or



- Find the area of the orange face (b x l) and multiply by the height(h).

By exploring volume in this way learners can then begin to understand that the volume of triangular prisms and cylinders can be found by calculating the cross-sectional area and multiplying by the length.

Problems in the context of area and volume can provide learners with the opportunity to develop and demonstrate skills in reasoning and communicating solutions. Some example problems are given below:

Chocolate Cake

Toby is 2 tomorrow and he wants a big sticky chocolate cake for his party! The problem is that my recipe specifies a 20cm round tin with a depth of 7.5cm, but I haven't got a 20cm round tin.

I do have a 23cm round tin, and I've also got a square tin, which has a side length of 15cm and a depth of 6 cm.

Which one do you think would be best? Or should I go out and buy a new cake tin?

Source: https://nrich.maths.org/7465

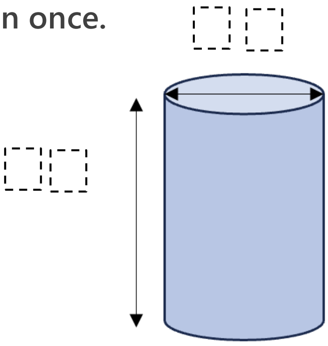
Which container

Learners investigate the dimensions of a cuboid shaped juice carton to find the most cost-effective design.
They could:

- Check that given examples have a capacity of 1 litre.
- Calculate the surface area for some given examples.
- Investigate to find the dimensions which minimise the surface area. A spreadsheet could be used to calculate and record solutions.
- Investigate other packaging designs.

Volume of a cylinder

Using the digits 0 to 9, place a digit in each box to give this cylinder the maximum volume possible. The same digit cannot be used more than once.



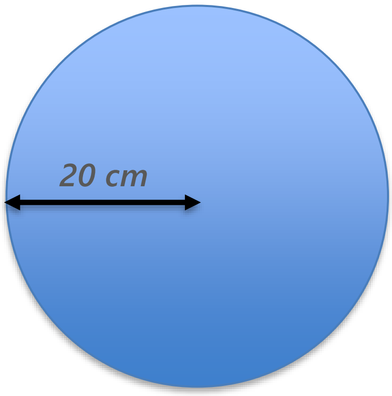
Source:openmiddle.com

Sheds

Estimate and then calculate the volume of the shed in cubic metres.

Accuracy

Problems set in the context of measure provide an excellent opportunity to practice and apply rounding and estimation skills. A common error when rounding to a number of significant figures is to fail to maintain the magnitude of the number. For example, rounding 48 230 to 5 or 50 instead of 50 000. Practising rounding within the context of measurement calculations can help to address this common error. Learners should be encouraged to explore the impact of premature rounding on the accuracy of their calculations and in general should be encouraged to round only at the final step. Tasks like the one below can be used to build learners' understanding of accuracy.



Area of a circle

Calculate the area of a circle and appreciate the effect of taking different values of π

Calculate the area of the circle:
(i) as a multiple of π
(ii) if π is taken as approximately 3
(iii) if π is taken as approximately 3.1
(iv) if π is taken as approximately 3.14
(v) if the button on your calculator is used

Which of the answers above is most accurate?

Points to consider:

- Learners can struggle when working with 2D representations of 3D objects. What prisms and cylinders are there readily 'to hand' in your classroom or department? What familiar objects could you easily present learners with to help them better understand surface area and volume work?
- Taking time to consider area and volume problems and investigate how the various formulae are derived and connected should prepare learners to solve a wide range of problems including those beyond fourth level in new and unfamiliar contexts.

Reflective questions:

- **How can we ensure that learners at fourth level continue to have the opportunity to carry out practical tasks?**

- **When calculating perimeter, area and volume how can we build on prior knowledge and understanding to investigate and derive formula for an extended range of 2D shapes and 3D solids.**
- **How can we use pictorial approaches to support understanding and to model problem solving strategies?**

Links across the curriculum:

Numeracy and Mathematics

- Estimation and rounding
  - estimation in the context of measure
  - rounding to a specified significant figure
  - demonstrates impact of inaccuracy and error
- Number – Interprets and solves multi-step problems.
- Fractions, decimal fractions, and percentages – problems in context of measure
- Money
  - best value
  - budgeting
- Properties of 2D shapes and 3D objects
  - Using Pythagoras' Theorem
  - Area and circumference of a circle – Details included in resource.
- Angle, symmetry, and transformation – similar figures
- Time – Carries out calculations involving speed, distance and time.

Technologies

- Food and Textile Technology - Practical skills and cooking techniques, e.g., weighing and measuring.
- Craft, Design, Engineering and Graphics - designing and constructing models/products.

Science

- makes accurate measurements using appropriate devices and units.

Education Scotland
Denholm House
Almondvale Business Park
Almondvale Way
Livingston EH54 6GA

T +44 (0)131 244 4330
E enquiries@educationscotland.gov.scot
www.education.gov.scot